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Risk Theory

Exercise Sheet 11

Due to: July 15, 2014

Note: Please submit exercise sheets in couples!

Problem 1 (6 credits)

Consider the individual model $X^{ind} = \sum_{i=1}^{n} U_i$, with $U_i \sim (1-\theta_i)\delta_0(\cdot) + \theta_i F_{V_i}(\cdot)$, $i = 1, \dots, n$. Show that in the case of the compound Poisson, binomial and negative binomial approximation it holds that

$$\mathbb{E}X^{ind} = \mathbb{E}X^{col} = \sum_{i=1}^{n} \theta_i \mathbb{E}V_i.$$

Problem 2 (6 credits)

Consider the individual model, $X^{ind} = \sum_{i=1}^{n} U_i$, with $U_i \sim (1-\theta_i)\delta_0(\cdot) + \theta_i F_{V_i}(\cdot)$, $i = 1, \dots, n$.

(a) Show that the variance of X^{ind} is given by

$$\operatorname{Var} X^{ind} = \sum_{i=1}^{n} \theta_i \mathbb{E}V_i^2 - \sum_{i=1}^{n} \theta_i^2 (\mathbb{E}V_i)^2.$$

(b) Show that in the case of the compound Poisson approximation and the compound binomial approximation it holds that

$$\operatorname{Var} X^{ind} \leq \operatorname{Var} X^{col}.$$

Problem 3 (6 credits)

Assume there is an algorithm simulating a random variable $X \sim \mathrm{U}([0,1])$. Describe with the help of the inverse method an algorithm generating

(a) a Pareto-distributed random variable with parameters $\alpha > 0$ and $c > 0$.

(b) a Weibull-distributed random variable with parameters $r > 0$ and $c > 0$.

Problem 4 (6 credits)

Let $X = \sum_{i=1}^{N} U_i$ be the total claim amount in the collective model, where $N \sim \mathrm{Poi}(\lambda)$, $\lambda > 0$ and $U_i \sim \mathrm{U}([0,1])$. The premium Π has to be chosen in a way such that $\Pi = \mathbb{E}[X + R]$, where R is the contribution restitution (Beitragsrückgewähr) defined as follows:

$$R = \begin{cases} \Pi/2 & \text{, if } N = 0 \\ (\Pi - U_1)/2 & \text{, if a single claim of size } U_1 \text{ has been reported} \\ 0 & \text{, if more than one claim has been reported.} \end{cases}$$

Compute Π.

Problem 5 (6 credits)

Let X be an Erlang distributed risk with parameters $n \in \mathbb{N}$ and $\lambda > 0$, that is $X \sim \mathrm{Erl}(n, \lambda)$. Let $\Pi(X)$ be the premium for X.

(a) Calculate the expected value and the variance of X.

(b) Calculate $\Pi(X)$ using the expected value principle, that is

$$\Pi(X) = (1 + a)\mathbb{E}X, \; a \geq 0.$$

(c) Calculate $\Pi(X)$ using the standard deviation principle, that is

$$\Pi(X) = \mathbb{E}X + a\sqrt{\mathrm{Var}\,(X)}, \; a \geq 0.$$