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# Risk Theory

## Exercise Sheet 12

Discussion on July 22, 2014

### Problem 1

A compound risk $X$ is modelled by the number $N$ of claims,

| | | | |
|---|---|---|---|
| $k$ | 0 | 1 | 2 |
| $\mathbb{P}(N=k)$ | 0.5 | 0.3 | 0.2 |

as well as by the claim amounts $U_i,\ i=1,2$.

| | | | |
|---|---|---|---|
| $k$ (in €) | 100 | 500 | 1000 |
| $\mathbb{P}(U_i=k)$ | 0.5 | 0.3 | 0.2 |

Let the random variables $N$, $U_1$ and $U_2$ be stochastically independent. The insurance company compensates for the first reported claim in full height, but only one third of the claim size of the second reported claim.

(a) Compute the expected value of $X$ in the case of the insurance holder reporting every claim.

(b) Compute the expected value of $X$ in the case where the insurance holder reports the first claim only if its amount is 500 or 1000. If the first claim is not reported, the second claim is fully covered (if it occurs).

### Problem 2

Let $W_1, W_2, \ldots$ be independent random variables having geometric distribution with parameter $p \in (0,1)$. Show that $T_n = W_1 + \ldots + W_n \sim \mathrm{NB}(n,p)$.

### Problem 3

Let $X_1 \sim \mathrm{NB}(n_1,p)$ and $X_2 \sim \mathrm{NB}(n_2,p)$ be independent random variables. Show that $X_1 + X_2 \sim \mathrm{NB}(n_1+n_2,p)$.

### Problem 4

Let $X > 0$ be a risk with distribution function $F$ and mean rest Hazard function $\mu_F(x)$ for all $x$ with tail probability function $\overline{F}(x) > 0$.

(a) Show that $\mathbb{E}\max(X-d,0) = \overline{F}(d)\,\mu_F(d),\ d>0$.

(b) Show that $\mathbb{E}X = \overline{F}(d)\,\mu_F(d) + \mathbb{E}[\min(X,d)],\ d>0$.

**Problem 5**

Consider two stochastically independent insurance portfolios (collective model). For the first portfolio, let the number of claims $N \sim \mathrm{Poi}(100)$ and the claim size $U_1 \sim \mathrm{LN}(10, 2)$. For the second portfolio, let the number of claims $L$ be such that $\mathbb{E}[L] = 1000$ and $\mathrm{Var}[L] = 1200$ and the claim size $Y_1 \sim \mathrm{Exp}(1/5000)$.

(a) Calculate the expected value and the variance of the total claim of each portfolio.

(b) Provide a lower bound for the capital of the insurance company to cover the sum of the total claim amount of both portfolios with a probability of 99% (use Tchebyshev's inequality).