Prof. Dr. E. Spodarev
Judith Olszewski

summer term 2014
May 13, 2014

Risk Theory

Exercise Sheet 3

Due to: May 20, 2014

Note: Please submit exercise sheets in couples!

Problem 1 (6 credits)

Compute the mean residual Hazard function $\mu_F(x)$ for $x \geq 0$ of the

(a) exponential distribution $\mathrm{Exp}(\lambda), \lambda > 0$.

(b) uniform distribution $\mathrm{U}[a,b], a < b$.

(c) Pareto distribution $\mathrm{Par}(\alpha, c), \alpha > 1, c > 0$.

Problem 2 (4 credits)

(a) Let $m(\cdot)$ be the hazard rate function of a cdf F. Prove that

$$\mu_F(x) = \int_x^\infty \exp\left(-\int_x^t m(y)dy\right) dt, x \geq 0.$$

(b) Let $x \geq 0$. Show that if $m(x) \geq \lambda$ then $\mu_F(x) \leq \frac{1}{\lambda}$ and that if $m(x) \leq \lambda$ then $\mu_F(x) \geq \frac{1}{\lambda}$.

(c) Let $x \geq 0$. Show that if $m(x) \geq \lambda$ then $\bar{F}(x) \leq \exp(-\lambda x)$ and that if $m(x) \leq \lambda$ then $\bar{F}(x) \geq \exp(-\lambda x)$.

Problem 3 (3 credits)

(a) Show that if X is a continuous risk with density f_X, then

$$\frac{1}{m(x)} = \int_0^\infty \frac{f_X(x+y)}{f_X(x)} dy.$$

(b) Check whether the gamma distribution $\Gamma(a, \lambda), a, \lambda > 0$ has a decreasing or an increasing hazard rate function.

Problem 4 (2 credits)

Let X be an absolutly continuous random variable with density f and $\lim_{x\to\infty} m(x)$ exists. Prove that F_X is heavy tailed if and only if $\lim_{x\to\infty} m(x) = 0$.

Problem 5 (9 credits)

Show that the following distributions are light-tailed:

(a) Normal distribution $\mathrm{N}(\mu, \sigma^2)$ with parameters $\mu \in \mathbb{R}$ and $\sigma^2 > 0$.

(b) Exponential distribution $\mathrm{Exp}(\lambda)$ with parameter $\lambda > 0$.

(c) Erlang distribution $\mathrm{Erl}(n, \lambda)$ with parameters $\lambda > 0$ and $n \in \mathbb{N}$.

(d) Distribution $\chi^2(n)$ with parameter $n \in \mathbb{N}$.

Show that the following distributions are heavy-tailed:

(e) Benktander type I distribution $\mathrm{BenI}(a, b, c)$ with parameters $a, b, c > 0$, where $a(a+1) \geq 2b$ and $ac \leq 1$.

(f) Benktander type II distribution $\mathrm{BenII}(a, b, c)$ with parameters $a, b, c > 0$, $a > 0, b \in (0,1)$ and $0 < c < a^{-1}e^{a/b}$.