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summer term 2014
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Risk Theory

Exercise Sheet 4

Due to: May 27, 2014

Note: Please submit exercise sheets in couples!

Problem 1 (3 credits)

Show that a real-valued stochastic process $\{X(t), t \geq 0\}$ with independent increments has stationary increments if for all $t \geq 0$ the distribution of the (one-dimensional) random variable $X(t+h) - X(h)$ does not depend on $h \geq 0$.

Problem 2 (9 credits)

Let $\{N(t), t \geq 0\}$ be a mixed Poisson process with mixing random variable Λ.

(a) Show that for the generating function $\hat{g}_{N(t)}(s) = \mathbb{E}s^{N(t)}, |s| < 1$ of $N(t)$ it holds that

$$\hat{g}_{N(t)}(s) = \hat{l}_\Lambda(t(1-s)),\ \forall t \geq 0.$$

(b) Show that

(1) $\mathbb{E}[N(t) \cdot (N(t)-1) \cdot \ldots \cdot (N(t)-k+1)] = t^k\, \mathbb{E}[\Lambda^k],\ k \in \mathbb{N}$,

(2) $\mathbb{E}[N(t)] = t \cdot \mathbb{E}[\Lambda]$,

(3) $\mathrm{Var}\,(N(t)) = t^2 \mathrm{Var}\,(\Lambda) + t\mathbb{E}[\Lambda]$.

Problem 3 (4 credits)

Let $\{N(t), t \geq 0\}$ be a Pascal process with parameters $a, b > 0$.

(a) Show that

$$\hat{g}_{N(t)}(s) = \left(\frac{b}{b + t(1-s)}\right)^a, s \in (-1,1), t \geq 0.$$

(b) Show that

$$p_k(t) = \binom{a+k-1}{k} \left(\frac{t}{t+b}\right)^k \left(\frac{b}{t+b}\right)^a$$

i.e. $N(t)$ follows a negative binomial distribution with parameters a and $\frac{t}{t+b}$.

Problem 4 (4 credits)

Let $\{N(t), t \geq 0\}$ be a Delaporte process. The density of the mixing random variable is given by

$$f_\Lambda(\lambda) = \frac{\eta^a (\lambda - b)^{a-1}}{\Gamma(a)} e^{-\eta(\lambda-b)} \mathbb{I}(\lambda > b).$$

(a) Show that

$$p_0(t) = e^{-bt} \left(\frac{\eta}{\eta + t} \right)^a.$$

(b) Show that

$$\hat{g}_{N(t)}(s) = e^{-(1-s)tb} \left(\frac{\eta}{\eta + (1-s)t} \right)^a, |s| < 1.$$

Problem 5 (4 credits)

Determine the expected value, the variance and the index of dispersion of the negative binomial distribution with parameters $\alpha > 0$ and $p \in (0, 1)$.