Prof. Dr. E. Spodarev
Judith Olszewski

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Risk Theory

Exercise Sheet 5

Due to: June 3, 2014

Note: Please submit exercise sheets in couples!

Problem 1 (6 credits)

(a) Show that a stochastic process $\{X(t), t \geq 0\}$ with independent increments is a Markov process.

(b) Show that a stochastic process $\{X(t), t \geq 0\}$ with independent and stationary increments is a homogeneous Markov process.

Problem 2 (6 credits)

Show that for a mixed Poisson process $\{N(t), t \geq 0\}$ with mixing random variable Λ it holds that the density of the claim arrival times σ_n is

$$f_{\sigma_n}(x) = \int_0^\infty \lambda e^{-\lambda x} \frac{(\lambda x)^{n-1}}{(n-1)!} dF_\Lambda(\lambda) = \frac{n}{x}\, p_n(x),\ x > 0,$$

where $p_k(t) = P(N(t) = k),\ \forall t \geq 0, \forall k \in \mathbb{Z}_+$ and that

$$\hat{l}_{\sigma_n}(s) = \int_0^\infty \left(\frac{\lambda}{\lambda + s}\right)^n dF_\Lambda(\lambda),\ s > -\lambda,$$

$$\mathbb{E}\sigma_n = n\, \mathbb{E}\left[\frac{1}{\Lambda}\right], \mathrm{Var}\,(\sigma_n) = n^2 \mathrm{Var}\left(\frac{1}{\Lambda}\right) + n\, \mathbb{E}\left[\frac{1}{\Lambda^2}\right].$$

Problem 3 (6 credits)

Show that for a mixed Poisson process with mixing random variable Λ it holds that the density of the inter-arrival times T_n is

$$f_{T_n}(x) = \int_0^\infty \lambda e^{-\lambda x} dF_\Lambda(\lambda) = \frac{p_1(x)}{x},\ x > 0$$

and that

$$\mathbb{E}T_n = \mathbb{E}\left[\frac{1}{\Lambda}\right], \mathrm{Var}\,(T_n) = \mathrm{Var}\left(\frac{1}{\Lambda}\right) + \mathbb{E}\left[\frac{1}{\Lambda^2}\right].$$

Problem 4 (6 credits)

Consider a bonus-malus system with one malus class M and two bonus classes B_1 and B_2. The insurance collective may consist of two types of risks, R_1 and R_2, respectively, which differ only by the distribution of their claim numbers. The table shows probabilities for the number of claims per year. If a claim-free year occurs, the policy holder is reclassified into the superior class in the following year. If he happens to be in class B_2 already, he remains in that class.

Risk Type	**Number of claims**		
	1	2	3
R_1	0.7	0.2	0.1
R_2	0.5	0.3	0.2

If one claim or two claims occur, the insurance holder is downgraded (no matter what the claim size is) one class or two classes, respectively, in the following year. However, the maximal class to be downgraded to is class M. The expected size of a claim is 500 EUR. All claims occur independently. In exercise (b) and (c), we assume that a policy holder starts in class M in the first year.

(a) Compute the transition probability matrices for the two risks R_1 and R_2.

(b) What is the probability of a policy holder of risk type R_2 to be in class B_1 after 3 years?

(c) Assume that a policy holder is chosen randomly from a portfolio where 60% of all policies are of risk type R_1 and 40% of all policies are of risk type R_2. What is the probability of this policy holder to be in class B_1 after 2 years?

Problem 5 (6 credits)

Consider a bonus-malus system with 3 classes A, B, and C and classification diagram given in above Table. For the number of claims N per year we have $P(N=0)=\frac{5}{9}$, $P(N=1)=\frac{1}{9}$, and $P(N>1)=\frac{1}{3}$.

	A	B	C
A	0	1	>1
B	0	1	>1
C	$-$	0	>0

(a) What is the probability of a policy holder to be in class A after 2 years under the assumption that he is in class A at the beginning?

(b) Use the transition probability matrix $P=(p_{ij})_{i,j\in A,B,C}$ in order to compute the stationary distribution $(p(A),p(B),p(C))$, i.e. the solution of the system of equations $(p(A),p(B),p(C))P=(p(A),p(B),p(C))^T$.

(c) How do you have to choose $P(N=0), P(N=1)$, and $P(N>1)$ in order to get that $p(A)=p(B)=p(C)$?