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Risk Theory

Exercise Sheet 6

Due to: June 10, 2014

Note: Please submit exercise sheets in couples!

Problem 1 (4 credits)

Consider a bonus-malus system with classes P (perfect), A (average), B (bad), and D (dangerous). Given are the following transition rules.

1. From P to P if 0 claims occur or 1 claim occurs, to A if 2 claims occur, to B if 3 claims occur and to D if more than 3 claims occur.
2. From A to P if 0 claims occur, to A if 1 claim occurs, to B if 2 claims occur and to D if more than 2 claims occur.
3. From B to A if 0 claims occur, to B if 1 claim occurs, and to D if more than 1 claim occurs.
4. From D to B if 0 claims occur and to D if more than 0 claims occur.

(a) Construct the transition matrix using the probabilities 0.6 that 0 claims occur, 0.2 that 1 claim occurs, 0.1 that 2 claims occur and 0.1 that 3 claims occur.

(b) Compute the vector $(p(A), p(B), p(C), p(D))$ of stationary probabilities.

Problem 2 (5 credits)

For a portfolio of fire insurance contracts on buildings, the following data is given.

k	Number of policies with k claims
0	103705
1	11632
2	1767
3	255
4	44
5	6
6	2
≥ 7	0
	$\Sigma = 117411$

(a) Estimate the expected value of the number of claims N (per policy) by the sample mean λ_1 and the variance of N by the sample variance λ_2. Is the Poisson distribution an appropriate model for the number of claims?

(b) In the following, the result of part (a) will be further analyzed. For this purpose, N is modelled by a Poisson distribution with parameters λ_1 and λ_2, respectively. Compare both distributions with the empirical distribution of the number of claims.

(c) Use λ_1 and λ_2 to approximate the distribution of the number of claims by a negative binomial distribution and compare the results to those of part (b).

Problem 3 (4 credits)

Show that for $N \sim \text{Log}(p)$, $p \in (0,1)$, the formula $p_k = \left(a + \frac{b}{k}\right) \cdot p_{k-1},\ k = 2, 3, \ldots$ holds true for $a = p$ and $b = -p$.

Problem 4 (5 credits)

Show that for $N_1 \sim \text{Eng}(\theta, p)$ and $N_2 \sim \text{Log}(p)$ it holds that

$$\mathbb{P}(N_1 = k) \to \mathbb{P}(N_2 = k),\ k \in \mathbb{N},\ \theta \to 0$$

and thus $N_1 \xrightarrow{d} N_2,\ \theta \to 0$.

Problem 5 (6 credits)

Show that Kolmogorov's backward equations are valid.