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Risk Theory

Exercise Sheet 7

Due to: June 17, 2014

Note: Please submit exercise sheets in couples!

In all problems, please use the collective model of risk theory.

Problem 1 (6 credits)

Show that the Laplace transform of the total claim amount $X = \sum_{i=1}^{N} U_i$ is given by

$$\hat{l}_X(s) = \hat{g}_N(\hat{l}_{U1}(s)), \ s \in \mathbb{R}_+,$$

where $\hat{g}_N(z)$ is the generating function of N and $\hat{l}_{U1}(s)$ is the Laplace transform of U_1.

Problem 2 (6 credits)

Consider the total claim amount $X = \sum_{i=1}^{N} U_i$, where $N \sim \text{Poi}(\lambda)$, $\lambda > 0$ and $U_1 \sim \text{Exp}(a)$, $a > 0$. Show that for the Laplace transform it holds that

$$\hat{l}_X(s) = \exp\left(-\frac{\lambda\, s}{a+s}\right), \ s \in \mathbb{R}_+.$$

Problem 3 (6 credits)

Compute the distribution function and the density function of the continuous component of the compound

(a) Poisson distribution,

(b) geometrical distribution,

if $U_1 \sim \text{Exp}(a)$ for $a > 0$.

Problem 4 (6 credits)

Let $X = \sum_{i=1}^{N} U_i$ be a compound risk with $\mathbb{E}X^2 < \infty$, $\mathbb{E}U_1^2 < \infty$, $\mathbb{E}N^2 < \infty$. Show that

(a) $\mathbb{E}X = \mathbb{E}N \cdot \mathbb{E}U_1$,

(b) $\text{Var}\,(X) = \text{Var}\,(N) \cdot (\mathbb{E}U_1)^2 + \mathbb{E}N \cdot \text{Var}\,(U_1)$.

Problem 5 (6 credits)

Consider again the fire insurance on buildings of sheet 6. The following two tables provide information about the number of claims and claim sizes.

(a) Estimate the expected value and the variance of the total claim amount X.

(b) Determine the reserve capital that is required such that the total claim amount is covered with a probability of at least 95%. Use Tchebyshev's inequality for this purpose.

k	Number of policies with k claims
0	103705
1	11632
2	1767
3	255
4	44
5	6
6	2
≥ 7	0
	$\Sigma = 117411$

Claim size in the interval (in 100 €)	Number of claims	Average claim size (in 100 €)
$(0, 50]$	51	39
$(50, 100]$	118	72
$(100, 150]$	115	120
$(150, 200]$	77	179
$(200, 300]$	204	249
$(300, 500]$	583	399
$(500, 800]$	1278	647
$(800, 1000]$	818	898
$(1000, 2000]$	3569	1401
$(2000, 5000]$	6056	3009
$(5000, 10000]$	2162	6729
$(10000, 20000]$	807	13511
$(20000, 50000]$	251	27590
$(50000, 100000]$	60	69426
	$\Sigma = 16149$	