Risk Theory
Exercise Sheet 8
Due to: June 24, 2014

Note: Please submit exercise sheets in couples!

Problem 1 (6 credits)

Let $X = \sum_{i=1}^{N} U_i$ be a Poisson compound risk with $\mathbb{E} U_i^2 < \infty$. Prove the following central limit theorem:

$$\frac{X - \mathbb{E}[X]}{\sqrt{\text{Var}(X)}} \overset{d}{\to} \mathcal{N}(0, 1), \ \lambda \to \infty.$$ 

Problem 2 (6 credits)

Let $X = \sum_{i=1}^{N} U_i$ be the aggregate claim amount in the collective model, where $N \sim \text{Geo}(p)$, $p \in (0, 1)$ and $U \sim \text{Exp}(\delta)$, $\delta > 0$. Let $\bar{F}_X(x) = \mathbb{P}(X \geq x)$.

(a) Show that $\bar{F}_X(x) = p \exp(-(1-p)\delta x)$, $x > 0$.

(b) Determine the net stop-loss premium of the reinsurer if the retention limit of the primary insurer is $b > 0$.

Problem 3 (6 credits)

Show the equivalence of the statements a) and b) for two random variables $X$ and $Y$.

(a) $\int_{x}^{\infty} \bar{F}_X(t)dt \leq \int_{x}^{\infty} \bar{F}_Y(t)dt$, $\forall x \in \mathbb{R}$.

(b) $\mathbb{E}\max\{X, x\} \leq \mathbb{E}\max\{Y, x\}$, $\forall x \in \mathbb{R}$.

Problem 4 (6 credits)

Suppose that for the aggregate claim amount $X = \sum_{i=1}^{N} U_i$ in a collective model there is an interval with zero aggregate probability, i.e. $\mathbb{P}(a < X < b) = 0$, $a < b$. Show that for $a \leq d \leq b$,

$$\mathbb{E}[(X - d)_+] = \frac{b - d}{b - a} \mathbb{E}[(X - a)_+] + \frac{d - a}{b - a} \mathbb{E}[(X - b)_+]$$

That is, the net stop-loss premium can be calculated via linear interpolation.
Problem 5 (6 credits)

The following data is given for a portfolio of insurance contracts.

<table>
<thead>
<tr>
<th>Claim amount ( \sim \text{Exp}(\lambda), \lambda = \frac{1}{500} )</th>
<th>Number of policies</th>
<th>Probability of 1 claim</th>
<th>Probability of no claims</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>3</td>
<td>0.025</td>
<td>0.975</td>
</tr>
</tbody>
</table>

All claims are mutually independent. The insurance company uses the standard deviation principle to calculate the premium, i.e. \( \Pi(X) = \mathbb{E}[X] + K \cdot \sqrt{\text{Var}(X)}, \ K > 0 \). Assume that the initial capital \( u \) is equal to 0. Which of the following values of \( K \) does guarantee the solvency of the portfolio with probability 0.95?

\[ K = 0.5, \ K = 0.7, \ K = 0.9. \]

Show all of your calculations.