Prof. Dr. E. Spodarev
Judith Olszewski

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Risk Theory

Exercise Sheet 8

Due to: June 24, 2014

Note: Please submit exercise sheets in couples!

Problem 1 (6 credits)

Let $X = \sum_{i=1}^{N} U_i$ be a Poisson compound risk with $\mathbb{E}U_i^2 < \infty$. Prove the following central limit theorem:

$$\frac{X - \mathbb{E}[X]}{\sqrt{\operatorname{Var}(X)}} \xrightarrow{d} \mathrm{N}(0,1), \ \lambda \to \infty.$$

Problem 2 (6 credits)

Let $X = \sum_{i=1}^{N} U_i$ be the aggregate claim amount in the collective model, where $N \sim \mathrm{Geo}(p)$, $p \in (0,1)$ and $U \sim \mathrm{Exp}(\delta)$, $\delta > 0$. Let $\bar{F}_X(x) = \mathbb{P}(X \geq x)$.

(a) Show that $\bar{F}_X(x) = p\exp(-(1-p)\delta x)$, $x > 0$.

(b) Determine the net stop-loss premium of the reinsurer if the retention limit of the primary insurer is $b > 0$.

Problem 3 (6 credits)

Show the equivalence of the statements a) and b) for two random variables X and Y.

(a) $\int_x^\infty \bar{F}_X(t)dt \leq \int_x^\infty \bar{F}_Y(t)dt, \ \forall x \in \mathbb{R}$.

(b) $\mathbb{E}\max\{X, x\} \leq \mathbb{E}\max\{Y, x\}, \ \forall x \in \mathbb{R}$.

Problem 4 (6 credits)

Suppose that for the aggregate claim amount $X = \sum_{i=1}^{N} U_i$ in a collective model there is an interval with zero aggregate probability, i.e. $\mathbb{P}(a < X < b) = 0, \ a < b$. Show that for $a \leq d \leq b$,

$$\mathbb{E}[(X-d)_+] = \frac{b-d}{b-a}\mathbb{E}[(X-a)_+] + \frac{d-a}{b-a}\mathbb{E}[(X-b)_+].$$

That is, the net stop-loss premium can be calculated via linear interpolation.

Problem 5 (6 credits)

The following data is given for a portfolio of insurance contracts.

Claim amount	Number of policies	Probability of 1 claim	Probability of no claims
$\sim \text{Exp}(\lambda), \lambda = \frac{1}{500}$	3	0.025	0.975

All claims are mutually independent. The insurance company uses the standard deviation principle to calculate the premium, i.e. $\Pi(X) = \mathbb{E}[X] + K \cdot \sqrt{\text{Var}(X)},\ K > 0$. Assume that the initial capital u is equal to 0. Which of the following values of K does guarantee the solvency of the portfolio with probability 0.95?

$$K = 0.5,\ K = 0.7,\ K = 0.9.$$

Show all of your calculations.