Prof. Dr. E. Spodarev
Judith Olszewski

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Risk Theory

Exercise Sheet 9

Due to: July 1, 2014

Note: Please submit exercise sheets in couples!

Problem 1 (6 credits)

(a) Let $X, Y > 0$ be arbitrary risks (not necessarily stochastically independent). Which connection exists between "$cv(X+Y)$" and "$cv(X) + cv(Y)$", where "cv" denotes the coefficient of variation?

(b) Show that

$$cv(aX) = cv(X)$$

for any arbitrary risk X and for $a > 0$.

Problem 2 (6 credits)

Consider two risks X and Y, where their respective distributions are given by

- $\mathbb{P}(X = 0) = 0.9$ and $\mathbb{P}(X > t) = 0.1 \ \exp(-t)$.
- $\mathbb{P}(Y = 0) = 0.8$ and $\mathbb{P}(Y > t) = 0.2 \ (1 + t)^{-3}$.

(a) Show that both risks have the same expectation and variance.

(b) Compare the two risks using the semivariance

$$\mathbb{E}[(X - \mathbb{E}[X])_+]^2 = \mathbb{E}[X - \mathbb{E}[X]]^2 \mathbb{1}(X > \mathbb{E}[X]),$$

and decide which of these risks is the more dangerous one.
Hint: You may use $\int_{0.1}^{\infty} (x - 0.1)^2 \frac{3}{(1+x)^4} dx = \frac{10}{11}$.

Problem 3 (6 credits)

Let F be a continuous distribution function on $\mathbb{R}$ and assume that $\int_{-\infty}^{\infty} |x| dF(x) < \infty$. Show that the following holds for all $a \in \mathbb{R}$:

(a) $\int_{-\infty}^{\infty} (x - a)_+ dF(x) = \int_a^{\infty} \bar{F}(x) dx$.

(b) $\int_{-\infty}^{\infty} (a - x)_+ dF(x) = \int_{-\infty}^{a} F(x) dx$.

Problem 4 (6 credits)

Show the so-called one-cut-criterion. Let X and Y be two random variables with continuous distribution functions F_X and F_Y, respectively. Suppose that $E|X| < \infty$ and $\mathbb{E}|Y| < \infty$ and that $\mathbb{E}[X] \leq \mathbb{E}[Y]$. If for some $t_0 \in \mathbb{R}$ $F_X(t) \leq F_Y(t)$ for $t < t_0$ and $F_X(t) \geq F_Y(t)$ for $t > t_0$, then $X \leq_{sl} Y$.
Hint: You may use the following without proof. Let $h : \mathbb{R} \to \mathbb{R}$ be a measurable function with $\int_{-\infty}^{\infty} |h(t)|dt < \infty$ and $\int_{-\infty}^{\infty} h(t)dt \geq 0$. If for $t_0 \in \mathbb{R}$ it holds that $h(t) \leq 0$ for all $t < t_0$ and that $h(t) \geq 0$ for all $t > t_0$, then $\int_x^{\infty} h(t)dt \geq 0$ for all $x \in \mathbb{R}$.

Problem 5 (6 credits)

Let $X \sim \text{Bin}(1, p)$, $p \in (0, 1)$ and $Y \sim \text{Poi}(p)$. Apply the one-cut-criterion in order to show

$$X \leq_{sl} Y.$$