Dr. Tim Brereton
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Summer Term 2015

Stochastic Simulation

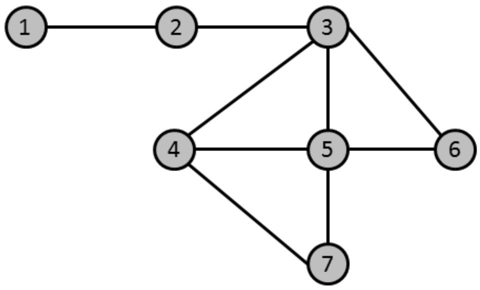
Problem Sheet 3

Deadline: May 7, 2015 at noon before the exercises

Please email your code to lisa.handl@uni-ulm.de AND hand in a printed copy of the code!

Exercise 1 (theory) (3 points)

Consider a random walk on the following graph.



Calculate the expected return time, $\mathbb{E}_i \tau_i^F$, to state i for each $i \in \mathcal{X}$.

Exercise 2 (theory) (5 points)

A total of N people live in two neighboring cities, A and B. For some strange reason, exactly one person each month gets fed up with his or her city and moves to the neighbouring city. No one else moves. People are more likely to get sick of a city if there are more people in it. More precisely, consider the number of people in city A. Call this number k. Each month, the probability that someone moves from city A to the neigbouring city is $\frac{k}{N}$. The probability someone moves from city B to city A is the complementary probability, $\frac{N-k}{N}$.

Compute the stationary distribution of the Markov chain $(X_n)_{n\in\mathbb{N}}$ describing the number of people living in city A by solving the detailed balance equations.

Exercise 3 (theory) (2 + 2 points)

Consider a Markov chain $\{X_n\}_{n\in\mathbb{N}}$ with transition matrix P and stationary distribution $\boldsymbol{\pi}$. Show that

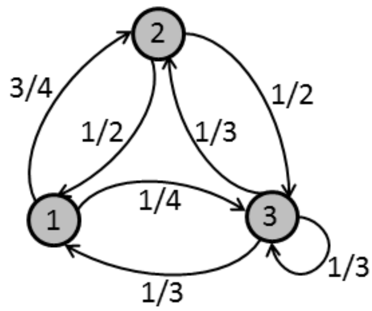
a) $\frac{P+\widehat{P}}{2}$ and

b) $\widehat{P}P$

are stochastic matrices and invariant for $\boldsymbol{\pi}$, where $\widehat{P}$ is the transition matrix of the time-reversal of $\{X_n\}_{n\in\mathbb{N}}$.

Exercise 4 (theory) (4 points)

Consider a Markov chain $\{Y_n\}_{n\in\mathbb{N}}$ with the following transition graph.



Is this Markov chain reversible? Provide a proof of your answer.

Exercise 5 (programming) (3 points)

Write a Matlab program to simulate the Markov chain $\{X_n\}_{n\in\mathbb{N}}$ defined in Exercise 2 for $N = 30$. Use the initial distribution $\boldsymbol{\mu_0} = \boldsymbol{\delta_0}$ and plot a histogram of the first 100 states, the first 1000 states and the first 10000 states.

Exercise 6 (programming) (3 + 1 points)

Consider the Markov chain $\{Y_n\}_{n\in\mathbb{N}}$ defined in Exercise 4, started from its invariant distribution π.

a) Write a Matlab program to simulate this Markov chain *backwards* from $n = 50$ to $n = 0$ and plot a realization of it.

b) Repeat the simulation in a) from (at least) $n = 10^5$ to $n = 0$ and estimate the transition probabilities P of the forward Markov chain $\{Y_n\}_{n\in\mathbb{N}}$.