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Summer Term 2015

Stochastic Simulation

Problem Sheet 8

Deadline: June 25, 2015 at noon before the exercises

Please email your code to lisa.handl@uni-ulm.de AND hand in a printed copy of the code!

Exercise 1 (theory) (5 points)

Recall that for a Markov chain $\{X_n\}_{n\in\mathbb{N}}$ on $\mathcal{X}$ with transition matrix P we defined

$$d(n) = \max_{i\in\mathcal{X}}\{\|\delta_i P^n - \pi\|_{TV}\}$$
$$\bar{d}(n) = \max_{i,j\in\mathcal{X}}\{\|\delta_i P^n - \delta_j P^n\|_{TV}\}$$

Show that $d(n) \leq \bar{d}(n) \leq 2d(n)$ holds for all $n \in \mathbb{N}$.

Exercise 2 (theory) (1 + 2 + 2 points)

In the lecture you studied how the Swendsen-Wang algorithm can be applied to simulate from the Ising model. Now we want to extend this approach to the simplified Pott's model as defined in Exercise 4 on Problem Sheet 6 for $J > 0$.

a) Write out the joint distribution of the random field X given by the Pott's model and the random variables U introduced by the Swendsen-Wang algorithm.

b) Show that the marginal distribution of X is the distribution defined by the Pott's model.

c) Write out (or describe in words) the conditional distributions of U given X and of X given U, respectively.

Exercise 3 (programming) (2 + 3 + 2 points)

Consider a Boltzmann distribution on $\{1,\ldots,20\}$ with energy function

$$\mathcal{E}(x) = -\exp\left(-\frac{(x-3)^2}{2}\right) - \exp\left(-(x-7)^2\right) - \exp\left(-\frac{(x-14)^2}{4}\right).$$

We want to draw from this distribution for temperature $T = 0.1$ using MCMC.

a) Write a Matlab program to sample from this model using the Metropolis algorithm and generate proposals by adding or subtracting 1 from the current value (each with probability 0.5). If you would leave $\{1,\ldots,20\}$ like this, stay in 1 or 20, respectively.

b) Extend your algorithm from a) such that you can use parallel tempering. Use $\beta = 0.1$ and the temperatures $T_1 = 0.1$, $T_2 = 0.3$ and $T_3 = 0.6$.

c) Run your programs from a) and b) at least $N = 10^5$ times and plot histograms of the theoretical distribution, the empirical distribution obtained using a) and the empirical distribution obtained using b).

Exercise 4 (programming) (5 + 2 points)

Consider the simplified Pott's model on a 10×10 lattice with periodic boundary conditions as defined in Exercise 4 on Problem Sheet 6.

a) Write a Matlab program to simulate from this model using the Swendsen-Wang algorithm.

b) Run your program for at least $N = 10^4$ steps and estimate the expected sum of all values of the random field. Remember the most frequent value in every iteration and draw a histogram in the end.