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Summer Term 2015

Stochastic Simulation

Problem Sheet 10

Deadline: July 9, 2015 at noon before the exercises

Please email your code to lisa.handl@uni-ulm.de AND hand in a printed copy of the code!

Exercise 1 (theory) (2 points)

Suppose you work at a call center and calls arrive according to a Poisson process, $\{N_t\}_{t\geq 0}$ with rate $\lambda = 10$ (t in hours). Calculate the expectation and the variance of the number of calls which arrive during the 8 hours you work there on a typical day.

Exercise 2 (theory) (2 + 2 points)

Consider a homogeneous Poisson process $\{N_t\}_{t\geq 0}$ with rate $\lambda = 2$.

a) Calculate the probability $P(N_2 \geq 5)$ and find $P(N_5 = 4 \mid N_3 = 2)$.

b) Find $P(N_4 - N_2 = 2 \mid N_3 = 4)$.

Exercise 3 (theory) (2 points)

Let $n \in \mathbb{N}$ and let $X_1, \ldots, X_n$ be iid. random variables with cumulative distribution function F. Express the cumulative distribution function of $X_{(1)}$ in terms of F.

Exercise 4 (theory) (5 points)

Let $\{N_t^{(1)}\}_{t\geq 0}$ and $\{N_t^{(2)}\}_{t\geq 0}$ be two independent homogeneous Poisson processes with positive intensities λ_1 and λ_2, respectively. Show that the sum of those processes, i.e.,

$$\{N_t^{(3)}\}_{t\geq 0} \quad \text{with} \quad N_t^{(3)} = N_t^{(1)} + N_t^{(2)} \quad \forall t \geq 0$$

is again a homogeneous Poisson process and determine its intensity.

Exercise 5 (programming) (4 + 2 points)

a) Write a Matlab program to simulate a homogeneous Poisson process with intensity $\lambda = 5$ in the interval $[0, 10]$

   1. by generating its interarrival times

2. by using a grid with a mesh size of $h = 0.05$

and plot one of its realizations (for each method).

b) Run your simulation programs from a) repeatedly (at least 10^4 times) and plot a histogram of the values of N_1 you obtain (relative frequencies) for each method and the pdf of its theoretical distribution.

Exercise 6 (programming) (3+1 points)

Consider the function

$$f(x) = \frac{1}{2}x^2 - 10 \exp\left(-\frac{x^2}{20}\right) \cos(3x).$$

a) Write a Matlab program to find the minimum of f using simulated annealing with geometric cooling. Start from $X_0 = 10$ and $T_0 = 10$ and use geometric cooling with $\beta = 0.99$. Draw proposals using a random walk sampler with normally distributed step sizes.

b) Run your program from a) several times and change the standard deviations of the step sizes (try $\sigma = 0.1$, 0.5 and 1). Use a sample size of at least $N = 10^3$. What do you observe?