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Stochastic Simulations
SoSe 2017
10. Juli 2017

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Exercise sheet 9 — till July 14, 2017

Theory (total – 9 points)

Exercise 9-1 (9 points)

Let (X, U) be a random point uniformly distributed on the area between some density $f(x)$ and the $x-$axis.
a) Find the joint density $g(x, u)$ of that point.
b) Show that the marginal density of X is $f(x)$.
c) Show that conditional densities $f_{X|U}$ and $f_{U|X}$ are proportional to the joint density $g(x, u)$. Determine the conditional distributions of X and U given that the respective other random variable is known.

Programming (total – 10 points)

Exercise 9-2 (2+2 points)

Consider the function

$$f(x) = \frac{1}{2}x^2 - 10\exp(-x^2)\sin(3x).$$

a) Write an R program to find the minimum of f using simulated annealing with geometric cooling. Start from $X_0 = 10$ and $T_0 = 10$ and use geometric cooling with $\beta = 0.98$. Draw proposals using a random walk sampler with normally distributed step sizes.
b) Run your program from a) several times and change the standard deviations of the step sizes (try $\sigma = 0.25, 0.5, 0.75$ and 1). Use a sample size of at least $N = 10^3$. What do you observe?

Exercise 9-3 (2+2+2 points)

Consider a Boltzmann distribution on $\{1, \ldots, 20\}$ with energy function

$$\mathcal{E}(x) = -\exp\left(-(x-3)^2/2\right) - \exp(-(x-7)^2) - \exp(-(x-14)^2/4).$$

We want to draw from this distribution for temperature $T = 0.5$ using MCMC.
a) Write an R program to sample from this model using the Metropolis algorithm and generate proposals by adding or subtracting 1 from the current value (each with probability 0.5). If you would leave $\{1, \ldots, 20\}$ like this, stay in 1 or 20, respectively.
b) Extend your algorithm from a) such that you can use parallel tempering. Use $\beta = 0.1$ and the temperatures $T_1 = 0.1, T_2 = 0.3$ and $T_3 = 0.6$.
c) Run your programs from a) and b) at least $N = 10^5$ times and plot histograms of the theoretical distribution, the empirical distribution obtained using a) and the empirical distribution obtained using b).