Stochastic Simulations
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Exercise sheet 6 — till June 9, 2017

Theory (total – 11 points)

Exercise 6-1 (3 points)

Prove that the Metropolis-Hastings algorithm works, i.e., prove that (for any proposal matrix Q) the transition matrix of the Markov chain used in the Metropolis-Hastings algorithm is in detailed balance with $\boldsymbol{\pi}$.

Exercise 6-2 (3 points)

Consider a Markov chain with state space $\mathcal{X}$ and transition matrix P, and two arbitrary probability measures, $\boldsymbol{\mu}$ and $\boldsymbol{\nu}$, on $\mathcal{X}$. Show that

$$\|\boldsymbol{\mu}P - \boldsymbol{\nu}P\|_{TV} \leq \|\boldsymbol{\mu} - \boldsymbol{\nu}\|_{TV}$$

and explain why this means that a Markov chain can only get closer to its stationary distribution, assuming that one exists.

Exercise 6-3 (5 points)

In Barker's algorithm for Markov Chains one takes the acceptance probability as

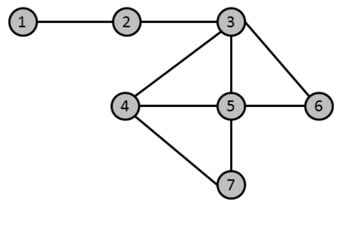
$$\alpha_{i,j} = \frac{\lambda_j}{\lambda_i + \lambda_j}.$$

Show that a Metropolis-Hastings algorithm with this acceptance probability generates a Markov Chain with the target distribution as stationary distribution under a certain condition (which ?) on matrix Q.

Programming (total – 8 points)

Exercise 6-4 (4 points)

Consider the following graph.



The graph is coloured by red and green such that no two neighboring vertices are red. Write the R code to generate the uniform sample ($n = 100$) from all possible appropriate colouring of this graph.

Exercise 6-5 (4 points)

Construct by the Metropolis algorithm the transition matrix P of a Markov chain with state space $E = \{0, 1 \ldots, l\}$ and limit distribution

$$\pi_i = \frac{\mu^i \exp(-\mu)}{z \cdot i!}, \quad \forall\, i \in E,$$

where $\mu \in (0, 1)$ and z is a normalizing constant of the probability mass function. A potential successor of state i should be chosen uniformly on $\{i - 1, i + 1\}$ (set $-1 = 0$, $l + 1 = l$).