Aufg. 8

$$P\left(\frac{X}{X+Y} \leq z\right) = P(X \leq z(X+Y)) = P((1-z)X \leq zY) = (*)$$

$$A = \{(t_1,t_2) \mid \underbrace{(1-z)t_1 \leq zt_2}_{\underset{z\neq 0}{\Leftrightarrow} \frac{1-z}{z}t_1 \leq t_2}\}$$

$$(*) = \int_A f_{X,Y}(t_1,t_2)\,dt_2\,dt_1 = \int_A e^{-t_1} e^{-t_2}\,dt_1\,dt_2 = \int_0^\infty \int_{\frac{1-z}{z}t_1}^\infty e^{-t_1} e^{-t_2}\,dt_2\,dt_1$$

$$= \int_0^\infty e^{-t_1} \left(-e^{-t_2}\right)\Big|_{\frac{1-z}{z}t_1}^\infty dt_1 = \int_0^\infty e^{-\frac{1}{z}t_1}\,dt_1 = z$$

Es gilt: $P\left(\frac{X}{X+Y} \leq z\right) = \begin{cases} 0, & z<0 \quad \text{, da } X,Y \geq 0 \text{ f.s.} \\ 1, & z>1 \quad \text{, da } Y \geq 0 \text{ f.s. und somit } \frac{X}{X+Y} \leq 1 \text{ f.s.} \end{cases}$

$$\Rightarrow P\left(\frac{X}{X+Y} \leq z\right) = \begin{cases} 0, & z<0 \\ z, & 0 \leq z \leq 1 \\ 1, & z>1 \end{cases}$$ Vert.fkt. der Gleichvert.

$$\Rightarrow \frac{X}{X+Y} \sim U[0,1]$$