<u>Aufg. 5</u>

$X, Y \sim U[0,1]$ unabh.

$\Longrightarrow f_{X,Y}(x,y) = f_X(x) \cdot f_Y(y) = \mathbb{1}_{[0,1]}(x)\ \mathbb{1}_{[0,1]}(y) \qquad , x, y \in \mathbb{R}$

$\Longrightarrow \mathbb{E}(XY) = \int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty} xy\, f_{X,Y}(x,y)\, dx\, dy = \int\limits_0^1 x\, dx \int\limits_0^1 y\, dy = \frac{x^2}{2}\Big|_0^1\ \frac{y^2}{2}\Big|_0^1 = \frac{1}{4}$

$\left(\underset{\substack{\uparrow \\ X,Y \text{ unabh.}}}{=} \mathbb{E}(X)\, \mathbb{E}(Y) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4} \right)$

$\mathbb{E}(XY)^2 = \int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty} (xy)^2 f_{X,Y}(x,y)\, dx\, dy = \int\limits_0^1 x^2\, dx \int\limits_0^1 y^2\, dy = \frac{1}{3} \cdot \frac{1}{3} = \frac{1}{9}$

$\Longrightarrow \mathrm{Var}(XY) = \mathbb{E}(XY)^2 - (\mathbb{E}(XY))^2 = \frac{1}{9} - \left(\frac{1}{4}\right)^2 = \frac{1}{9} - \frac{1}{16} = \cancel{\frac{7}{138}}\ \frac{7}{144} = 0.4861$

Aufg. 6

$$\varphi_{X_1+X_2}(t) \underset{\uparrow}{=} \prod_{i=1}^{2} \varphi_{X_i}(t) = e^{i\mu_1 t - \frac{1}{2}\sigma_1^2 t^2} \, e^{i\mu_2 t - \frac{1}{2}\sigma_2^2 t^2}$$

X_1, X_2 unabh.

$$= \underbrace{e^{i(\mu_1+\mu_2)t - \frac{1}{2}(\sigma_1^2+\sigma_2^2)t^2}}_{\text{charakt. Fkt. der } \mathcal{N}(\mu_1+\mu_2,\, \sigma_1^2+\sigma_2^2)\text{-Vat.}}$$