Aufg. 7

(a) $A_1 \Delta A_2 = (A_1 \cup A_2) \setminus (A_1 \cap A_2)$

Es gelte $\mathbb{P}(A_1 \Delta A_2) = 0$.

$\Rightarrow \mathbb{P}((A_1 \cup A_2) \setminus (A_1 \cap A_2)) = \mathbb{P}(A_1 \cup A_2) - \mathbb{P}(A_1 \cap A_2) = 0$

$\Rightarrow \mathbb{P}(A_1 \cup A_2) = \mathbb{P}(A_1 \cap A_2)$

Weil $A_1 \cup A_2 = (A_1 \setminus A_2) \uplus (A_1 \cap A_2) \uplus (A_2 \setminus A_1)$, gilt

$$\mathbb{P}(A_1 \setminus A_2) + \mathbb{P}(A_1 \cap A_2) + \mathbb{P}(A_2 \setminus A_1) = \mathbb{P}(A_1 \cap A_2)$$

$$\Rightarrow \mathbb{P}(A_1 \setminus A_2) = \mathbb{P}(A_2 \setminus A_1) \quad (*)$$

$$\Rightarrow \underline{\underline{\mathbb{P}(A_1)}} = \mathbb{P}((A_1 \setminus A_2) \uplus (A_1 \cap A_2)) = \mathbb{P}(A_1 \setminus A_2) + \mathbb{P}(A_1 \cap A_2)$$

$$\overset{(*)}{=} \mathbb{P}(A_2 \setminus A_1) + \mathbb{P}(A_1 \cap A_2) = \mathbb{P}((A_2 \setminus A_1) \uplus (A_1 \cap A_2))$$

$$= \underline{\underline{\mathbb{P}(A_2)}}$$

(b) $\mathbb{P}(A_1 \cap \dots \cap A_n) = 1 - \mathbb{P}((A_1 \cap \dots \cap A_n)^c) = 1 - \mathbb{P}(A_1^c \cup \dots \cup A_n^c) \geq 1 - \sum_{i=1}^{n} \mathbb{P}(A_i^c)$

$$= 1 - \sum_{i=1}^{n} (1 - \mathbb{P}(A_i)) = \mathbb{P}(A_1) + \dots + \mathbb{P}(A_n) - n + 1$$

$$= \mathbb{P}(A_1) + \dots + \mathbb{P}(A_n) - (n-1)$$