Aufgabe 4c – Lösungsvorschlag

$$\mathbb{P}(D(t) \geq x) = \int_0^t \mathbb{P}(T(t) \geq x-y \mid C(t)=y)\, \mathbb{P}_{C(t)}(dy)$$

$$= \mathbb{P}(C(t)=t)\, \mathbb{P}(T(t) \geq x-t \mid C(t)=t)\, \mathbb{I}\{x \geq t\}$$

$$+ \mathbb{P}(C(t) \geq x)\, \mathbb{I}\{x < t\}$$

$$+ \int_0^{\min\{x,t\}} \mathbb{P}(T(t) \geq x-y \mid C(t)=y)\, \mathbb{P}_{C(t)}(dy)$$

$$= e^{-\lambda t}\, \mathbb{P}(T_1 \geq x \mid T_1 > t)\, \mathbb{I}\{x \geq t\}$$

$$+ e^{-\lambda x}\, \mathbb{I}\{x < t\}$$

$$+ \int_0^{\min\{x,t\}} \mathbb{P}(T_{N(t)} \geq x \mid T_{N(t)} > y)\, \lambda e^{-\lambda y}\, dy$$

$$= e^{-\lambda t}\, \mathbb{P}(T_1 \geq x-t)\, \mathbb{I}\{x \geq t\} + e^{-\lambda x}\, \mathbb{I}\{x < t\} + \int_0^{\min\{x,t\}} \lambda e^{-\lambda x}\, dy$$

$$= e^{-\lambda t} e^{-\lambda(x-t)}\, \mathbb{I}\{x \geq t\} + e^{-\lambda x}\, \mathbb{I}\{x < t\} + \min\{x,t\}\, \lambda e^{-\lambda x}$$

$$= e^{-\lambda x}\left(1 + \lambda \min\{x,t\}\right)$$