Exercise 1
Write a computer program \texttt{int selfAvoiding(int n)} that computes the number of self-avoiding paths of length \( n \) starting in the origin

(a) if the underlying graph is the two-dimensional triangular lattice
(b) if the underlying graph is the two-dimensional hexagonal lattice.

Make sure that your implementation does not depend on the floating point arithmetic of the programming language!

Exercise 2
Let \( (a_n)_{n \geq 1} \) be a sequence of non-negative real numbers satisfying the condition \( a_{m+n} \leq a_m + a_n \) for all \( n, m \geq 1 \). Prove that \( \lim_{n \to \infty} a_n/n \) exists and equals \( \inf a_n/n \).

Hint: Let \( \gamma = \inf_{n \geq 1} a_n/n \). First prove that \( \gamma = \liminf_{n \to \infty} a_n/n \). For \( \varepsilon > 0 \) now choose \( k \geq 1 \) satisfying \( a_k/k \leq \gamma + \varepsilon \). Use the fact that any \( m \geq 1 \) can be written in the form \( nk + j \) to obtain suitable bounds on \( a_m/m \).

Exercise 3
Denote by \( \mu_n \) the number of self-avoiding walks on \( \mathbb{Z}^d \) of length \( n \) that start at the origin.

(a) show that \( \mu_{m+n} \leq \mu_m \mu_n \) holds for all \( m, n \geq 1 \).

(b) show that \( \kappa = \lim_{n \to \infty} (\mu_n)^{1/n} \) exists and satisfies the relation \( d \leq \kappa \leq 2d - 1 \).

Hint: Use problem 2 to prove the existence of the limit

Exercise 4
Consider Bernoulli bond-percolation on \( \mathbb{Z}^d \). Prove that \( \theta(p) \to 1 \) as \( p \to 1 \).