Prof. Dr. V. Schmidt
C. Hirsch

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Stochastic networks

Problem set 3

Due date: November 15, 2011

Exercise 1

Write a computer program **int** *selfAvoiding(***int** *n)* that computes the number of self-avoiding paths of length n starting in the origin

(a) if the underlying graph is the two-dimensional triagonal lattice

(b) if the underlying graph is the two-dimensional hexagonal lattice.

Make sure that your implementation does not depend on the floating point arithmetic of the programming language!

Exercise 2

Let $(a_n)_{n\geq 1}$ be a sequence of non-negative real numbers satisfying the condition $a_{m+n} \leq a_m + a_n$ for all $n, m \geq 1$. Prove that $\lim_{n\to\infty} a_n/n$ exists and equals $\inf a_n/n$.

Hint: Let $\gamma = \inf_{n\geq 1} a_n/n$. First prove that $\gamma = \liminf_{n\to\infty} a_n/n$. For $\varepsilon > 0$ now choose $k \geq 1$ satisfying $a_k/k \leq \gamma + \varepsilon$. Use the fact that any $m \geq 1$ can be written in the form $nk + j$ to obtain suitable bounds on a_m/m.

Exercise 3

Denote by μ_n the number of self-avoiding walks on $\mathbb{Z}^d$ of length n that start at the origin.

(a) show that $\mu_{m+n} \leq \mu_m \mu_n$ holds for all $m, n \geq 1$.

(b) show that $\kappa = \lim_{n\to\infty}(\mu_n)^{1/n}$ exists and satisfies the relation $d \leq \kappa \leq 2d - 1$.

Hint: Use problem 2 to prove the existence of the limit

Exercise 4

Consider Bernoulli bond-percolation on $\mathbb{Z}^d$. Prove that $\theta(p) \to 1$ as $p \to 1$.