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Stochastic networks
Problem set 4

Due date: November 22, 2011

Exercise 1

Write a computer program **int** *latticeAnimal(***int** *n)* that computes the number of connected subgraphs of $\mathbb{Z}^d$ containing the origin and consisting of precisely n vertices.

Bonus. Can you optimize your program so that it needs at most a^n steps for some $a \geq 1$?

Exercise 2

Let $n \geq 1$ and consider the subgraph $G_n \subset \mathbb{Z}^d$ induced by the vertex set $V = [0,n]^{d-1} \times \mathbb{Z}$. Prove that $p_{ec} = p_c = 1$.

Exercise 3

Construct a locally finite, countable and connected graph satisfying

(a) $p_c^s = 1$ and $p_c^b = 0$.

(b) $0 < p_c^s = p_c^b < 1$.

Hint. For problem (a) consider the following modification of the nearest-neighbor graph on $\mathbb{N}$. Replace each edge of the form $\{n-1, n\}$ by a subgraph consisting of n extra vertices $x_{n,1}, \ldots, x_{n,n}$ and for each $1 \leq j \leq n$ add edges between $n-1$ and $x_{n,i}$ and between $x_{n,i}$ and n. Observe that this graph does not have a uniform bound on vertex degrees. You may use without proof that for $p_1, p_2, \ldots \in (0,1)$ we have $\prod_{i \geq 1}(1-p_i) > 0$ if $\sum_{i \geq 1} p_i < \infty$.

Exercise 4

Let G be the hypercubic lattice $\mathbb{Z}^d$. Prove that there exists $p > 0$ and $c > 0$ such that $\mathbb{P}_p(|C_o^b| > n) \leq 2\exp(-cn^{1/d})$ holds for all $n \geq 1$.

Bonus. Can you use the bonus part of problem 1 to obtain a bound of the form $2\exp(-cn)$?