Prof. Dr. Evgeny Spodarev
Dipl.-Math. Stefan Roth

WS 2012/2013

Stochastics II
Exercise Sheet 4

Due to: Wednesday, 14th of November 2012

Exercise 1 (4 Points)

Show the existence of a stochastic process $X = \{X(t),\ t \in T\}$ which fullfills simultaneously the following conditions:

(a) The second moment does not exist

(b) The variogram $\gamma(s,t)$ is finite for all $s, t \in T$

Exercise 2 (5 Points)

Let $\{T_n\}_{n\in\mathbb{N}}$ be a sequence of i.i.d random variables with $T_1 \sim Exp(\lambda)$. The process $N = \{N(t),\ t \geq 0\}$ given by

$$N(t) := \sum_{n=1}^{\infty} 1\!\!1_{\{T_1+\cdots+T_n \leq t\}}$$

is called a Poisson-process with intensity λ.

(a) Prove: $N(t)$ is Poisson distributed for each $t > 0$.

(b) Determine the parameter of this Poisson distribution.

(c) Calculate $H(t) = EN(t)$.

Exercise 3 (4 Points)

Give examples for a stochastic process $X = \{X(t),\ t \in T\}$ with the following properties (with proof!):

(a) X has L_2-differentiable paths which are not a.s. differentiable.

(b) X has a.s. differentiable paths which are not L_1-differentiable.

Exercise 4 (3 Points)

Let $X = \{X(t),\ t \in [0,\infty)\}$ be a real-valued stochastic process with independent increments. Show that X has stationary increments if the distribution of $X(t+h)-X(h)$ does not depend on h for arbitrary $t \in [0,\infty)$.