Exercise 1 (4 Points)
Let $N = \{N(t), t \geq 0\}$ be a Cox process with intensity function $\lambda(t) = Z$, where $Z$ is a discrete random variable with
$$P(Z = \lambda_1) = P(Z = \lambda_2) = \frac{1}{2}.$$ Calculate the moment generating function, the expectation and the variance of $N(t)$.

Exercise 2 (4 Points)
Let $N^{(1)} = \{N^{(1)}(t), t \geq 0\}$ and $N^{(2)} = \{N^{(2)}(t), t \geq 0\}$ be two independent homogeneous Poisson processes with intensities $\lambda_1$ and $\lambda_2$. Furthermore let $X$ be an arbitrary non-negative random variable which is independent of $N^{(1)}$ and $N^{(2)}$. Show that the process $N$ defined by
$$N(t) = \begin{cases} N^{(1)}(t), & t \leq X \\ N^{(1)}(X) + N^{(2)}(t - X), & t > X \end{cases}$$ is a Cox process with intensity process $\lambda = \{\lambda(t), t \geq 0\}$ given by
$$\lambda(t) = \begin{cases} \lambda_1, & t \leq X \\ \lambda_2, & t > X \end{cases}$$

Exercise 3 (3 Points)
Give an algorithm of how to simulate the trajectories of a Wiener process $W = \{W(t), t \in [0, 1]\}$ by using the independence and the distribution of the increments of $W$.

Exercise 4 (5 Points)
Let $W = \{W(t), t \in [0, 1]\}$ be a Wiener process and $L := \arg\max_{t \in [0, 1]} W(t)$. Show that
$$P(L \leq x) = \frac{2}{\pi} \arcsin \sqrt{x}, \quad x \in [0, 1]$$

Hint: Use the fact $\max_{r \in [0,t]} W(r) \overset{d}{=} |W(t)|$. 

Stochastics II
Exercise Sheet 7
Due to: Wednesday, 5th of December 2012