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WS 2012/2013

Stochastics II
Exercise Sheet 7

Due to: Wednesday, 5th of December 2012

Exercise 1 (4 Points)

Let $N = \{N(t),\ t \geq 0\}$ be a Cox process with intensity function $\lambda(t) = Z$, where Z is a discrete random variable with

$$P(Z = \lambda_1) = P(Z = \lambda_2) = \frac{1}{2}$$

Calculate the moment generating function, the expectation and the variance of $N(t)$.

Exercise 2 (4 Points)

Let $N^{(1)} = \{N^{(1)}(t),\ t \geq 0\}$ and $N^{(2)} = \{N^{(2)}(t),\ t \geq 0\}$ be two independent homogeneous Poisson processes with intensities λ_1 and λ_2. Furthermore let X be an arbitrary non-negative random variable which is independent of $N^{(1)}$ and $N^{(2)}$. Show that the process N defined by

$$N(t) = \begin{cases} N^{(1)}(t) & ,\ t \leq X \\ N^{(1)}(X) + N^{(2)}(t - X) & ,\ t > X \end{cases}$$

is a Cox process with intensity process $\lambda = \{\lambda(t),\ t \geq 0\}$ given by

$$\lambda(t) = \begin{cases} \lambda_1 & ,\ t \leq X \\ \lambda_2 & ,\ t > X \end{cases}$$

Exercise 3 (3 Points)

Give an algorithm of how to simulate the trajectories of a Wiener process $W = \{W(t),\ t \in [0,1]\}$ by using the independence and the distribution of the increments of W.

Exercise 4 (5 Points)

Let $W = \{W(t),\ t \in [0,1]\}$ be a Wiener process and $L := \operatorname*{argmax}_{t \in [0,1]} W(t)$. Show that

$$P(L \leq x) = \frac{2}{\pi} \arcsin \sqrt{x}, \qquad x \in [0,1]$$

Hint: Use the fact $\max_{r \in [0,t]} W(r) \stackrel{d}{=} |W(t)|$.