Exercise 1 (3 Points)
Let \( W = \{W(t), t \geq 0\} \) be the Wiener process. Define
\[
Q(a, b) = P(\exists t \geq 0, W(t) > at + b)
\]
a, b > 0. \( Q(a, b) \) is the probability that \( W \) crosses the half-line \( y = at + b \). Show the following statements:

(a) \( Q(a, b) = Q(b, a) \) and \( Q(a, b_1 + b_2) = Q(a, b_1)Q(a, b_2) \)

(b) \( Q(a, b) = \exp(-2ab) \)

Exercise 2 (3 Points)
Let \( X_1, \ldots, X_n \) be a sequence of i.i.d. infinitely divisible (ID) random variables. Show that the random variable \( Y \) given by
\[
Y = \sum_{k=1}^{n} a_k X_k
\]
is ID as well for arbitrary \( a_1, \ldots, a_n \in \mathbb{R} \) if the Lévy measure of \( X_1 \) is absolutely continuous w.r.t. the Lebesgue measure on \( \mathbb{R} \). Calculate the Lévy characteristics of \( Y \).

Exercise 3 (6 Points)
Let \( X \) be a random variable with characteristic function \( \varphi \).

(a) Show: If \( X \) is ID \( \Rightarrow \) \( \varphi(z) \neq 0, \forall z \in \mathbb{R} \)

**Hint:** Show that \( \lim_{n \to \infty} \varphi_n(z) = 1 \) for all \( z \in \mathbb{R} \) and \( \varphi(z) = (\varphi_n(z))^n \). Use the fact that \( |\varphi_n|^2 \) is a characteristic function.

(b) Find an example for a random variable which is not ID (with proof).

Exercise 4 (4 Points)
Show that the function \( \varphi : \mathbb{R} \to \mathbb{C} \) given by
\[
\varphi(z) = \exp(\psi(z)), \quad \psi(z) = 2 \sum_{k=-\infty}^{\infty} 2^{-k}(\cos(2^k z) - 1)
\]
is the characteristic function of an ID random variable.

**Hint:** Use the Lévy-Kinchin representation with Lévy measure \( \nu(\{\pm 2^k\}) = 2^{-k}, k \in \mathbb{Z} \)