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Stochastics II
Exercise Sheet 2

Due to: Wednesday, 31st of October 2012

Exercise 1 (4 Points)

Give examples for a stochastic process $X = \{X(t),\ t \in T\}$ with the following properties (with proof!).

(a) X is not separable.

(b) X is stochastically continuous but not L_1-continuous.
Hint: Find a process which grows large on a contracting interval.

Exercise 2 (6 Points)

Let $W = \{W(t), t \in \mathbb{R}_+\}$ be a Wiener process. Which of the following processes are Wiener processes as well?

(a) $W_1 = \{W_1(t) := -W(t), t \in \mathbb{R}_+\}$,

(b) $W_2 = \{W_2(t) := \sqrt{t}W(1), t \in \mathbb{R}_+\}$,

(c) $W_3 = \{W_3(t) := W(2t) - W(t), t \in \mathbb{R}_+\}$.

Exercise 3 (3 Points)

Consider a stochastic process $X = \{X(t), t \in [0, 1]\}$ which consists of independent and identically distributed random variables with density $f(x)$, $x \in \mathbb{R}$. Show that such a process can not be **stochastically** continuous in $t \in [0, 1]$.

Exercise 4 (3 Points)

Show that the Poisson process is stochastically continuous although it does not possess any a.s. continuous modifications.