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WS 2012/2013
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Stochastics III - Exercise sheet 1

Due to: 24. 10. 2012 before exercises start

Exercise 1 (4 points)
Let $A = (a_{ij})_{1\le i,j\le n} \in \mathbb{R}^{n\times n}$ be a matrix with real eigenvalues $\lambda_1, \dots, \lambda_n$. Show that

$$tr(A) := \sum_{i=1}^{n} a_{ii} = \sum_{i=1}^{n} \lambda_i \ .$$

Exercise 2 (3+1 points)

(a) Let $A \in \mathbb{R}^{n\times m}$ with $rk(A) = r$. Show: If A has representation

$$A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix},$$

where A_{11} is nonsingular with $rk(A_{11}) = r$, then

$$A^- = \begin{pmatrix} A_{11}^{-1} & 0 \\ 0 & 0 \end{pmatrix} \in \mathbb{R}^{m\times n}$$

is a generalized inverse of A, i.e. $AA^-A = A$.

(b) Determine the generalized inverse of the matrix $A \in \mathbb{R}^{3\times 4}$,

$$A = \begin{pmatrix} 4 & 7 & -1 & 2 \\ 1 & 2 & 5 & -1 \\ 7 & 13 & 14 & -1 \end{pmatrix}.$$

Exercise 3 (4 points)
Let $A \in \mathbb{R}^{n\times n}$ be symmetric, $X = (X_1, \dots, X_n)$ an n-dimensional random vector with $\mathbb{E}X = \mu = (\mu_1, \dots, \mu_n)$ and covariance matrix $\Sigma = \mathbb{E}\left((X-\mu)(X-\mu)^\top\right)$. Show that

$$\mathbb{E}(X^\top AX) = tr(A\Sigma) + \mu^\top A\mu \ .$$

Exercise 4 (2+3 points)
Let $X = (X_1, \dots, X_n)$ be an n-dimensional random vector, where $\mathbb{E}(X_i^2) < \infty,\ 1 \le i \le n$. Furthermore, let $A \in \mathbb{R}^{m\times n}$. Show that

(a) $\mathbb{E}(AX) = A\,\mathbb{E}X$,

(b) $\text{Cov}(AX) = A\ \text{Cov}(X)\ A^\top$.