**Stochastics III - Exercise sheet 2**

Due to: 7. 11. 2012 before exercises start

**Exercise 1** (3 points)
Give an example of two random variables $X$ and $Y$ which are normally distributed but such that the vector $(X, Y)\top$ is not multivariate normally distributed.

**Exercise 2** (2 + 3 points)
Let $X = (X_1, X_2, X_3)\top \sim N(\mu, K)$ with expectation vector $\mu = (1, 2, 3)\top$ and covariance matrix

$$K = \sigma^2 \begin{pmatrix} 2 & \rho & 0 \\ \rho & 1 & \rho \\ 0 & \rho & 2 \end{pmatrix}.$$

(a) Determine the marginal distributions of $X_2$ und $(X_1, X_3)\top$.

(b) For which $\rho$ are the random variables $X_1 + X_2 + X_3$ and $X_1 - X_2 - X_3$ independent?

**Exercise 3** (4 + 4 points)
Proof Theorem 1.4 of the lecture course:
Let $Y$ be an $n$-dimensional random vector with expectation vector $\mu = EY$ and covariance matrix $K = \text{Cov}(Y)$, such that $\text{rk}(K) = r$ with $r \leq n$. The random vector $Y$ is normally distributed if and only if one of the following conditions is fulfilled.

(a) The characteristic function $\varphi(t) = \mathbb{E}\exp\left(i \sum_{j=1}^{n} t_j Y_j\right)$ of $Y$ is given by

$$\varphi(t) = \exp\left(it\top\mu - \frac{1}{2} t\top K t\right), \quad \forall t = (t_1, \ldots, t_n)\top \in \mathbb{R}^n.$$

(b) The linear function $c\top Y$ of $Y$ for every $c \in \mathbb{R}^n$ with $c \neq o$ is normally distributed with $c\top Y \sim N(c\top \mu, c\top K c)$.

**Exercise 4** (6 + 2 points)
Let $(X, Y)\top$ bivariate normally distributed with expectation vector $\mu = (1, 2)\top$ and covariance matrix

$$K = \begin{pmatrix} 5 & 2 \\ 2 & 3 \end{pmatrix}.$$

(a) Plot with R the density $f_{(X,Y)}$ of the random vector $(X, Y)\top$. Hint: Use the commands `dmvnorm`, `persp`.

(b) Calculate in R the expectation and the variance of the random variable $2X - Y + 3$. 