Stochastics II - Supplementation

Blatt 6, Aufgabe 1, (b)

$$\begin{aligned}
&\mathbb{P}[N_1 < 2, N_2 = 3, N_3 > 5] \\
&= \mathbb{P}[N_1 = 0, N_2 = 3, N_3 > 5] + \mathbb{P}[N_1 = 1, N_2 = 3, N_3 > 5] \\
&= \mathbb{P}[N_1 - N_0 = 0, N_2 - N_1 = 3, N_3 - N_2 > 2] + \mathbb{P}[N_1 - N_0 = 1, N_2 - N_1 = 2, N_3 - N_2 > 2] \\
&= \mathbb{P}[N_1 - N_0 = 0]\mathbb{P}[N_2 - N_1 = 3]\mathbb{P}[N_3 - N_2 > 2] + \mathbb{P}[N_1 - N_0 = 1]\mathbb{P}[N_2 - N_1 = 2]\mathbb{P}[N_3 - N_2 > 2] \\
&= \exp(-\lambda)\frac{\lambda^3}{6}\exp(-\lambda)\left(1 - \exp(-\lambda) - \lambda\exp(-\lambda) - \frac{\lambda^2}{2}\exp(-\lambda)\right) \\
&+ \lambda\exp(-\lambda)\frac{\lambda^2}{2}\exp(-\lambda)\left(1 - \exp(-\lambda) - \lambda\exp(-\lambda) - \frac{\lambda^2}{2}\exp(-\lambda)\right) \\
&= \left(1 - \exp(-\lambda) - \lambda\exp(-\lambda) - \frac{\lambda^2}{2}\exp(-\lambda)\right)\left(\exp(-\lambda)\frac{\lambda^3}{6}\exp(-\lambda) + \lambda\exp(-\lambda)\frac{\lambda^2}{2}\exp(-\lambda)\right) \\
&= \exp(-2\lambda)\frac{2\lambda^3}{3}\left(1 - \exp(-\lambda) - \lambda\exp(-\lambda) - \frac{\lambda^2}{2}\exp(-\lambda)\right)
\end{aligned}$$

Blatt 6, Aufgabe 4, (d)

$N_t + 1$ is a random variable, dependent of ξ. Thus, one cannot conclude that $\mathbb{E}\xi_{N_t+1} = \frac{1}{\lambda}$

$C(t)$ can take values between 0 and t. Therefore, $C(t) \overset{d}{=} \min\{t, E\}$, where $E \sim \text{Exp}(\lambda)$. As a consequence, for the expected value of $C(t)$ it holds that $\mathbb{E}C(t) > 0$ and $\mathbb{E}C(t) < \frac{1}{\lambda}$.

Blatt 4, Aufgabe 2, (b)

$1st$ Possibility
State 0 is recurrent, because the probability to have a 6 infinitely often is 1. The MC is irreducible, therefore, all states are recurrent.

$2nd$ Possibility

$$\sum_{n=1}^{\infty} p_{00}^{(n)} = \sum_{n=1}^{\infty} \frac{1}{6} = \infty.$$

$p_{00}^{(n)} = \frac{1}{6}$, because the probability to be in an arbitrary state after $n-1$ steps is 1 and to come back to 0 from that state is $\frac{1}{6}$. The MC is irreducible, therefore, all states are recurrent.