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Stochastics II

Exercise Sheet 1

Due: October 23, 2013

Note: Please submit exercise sheets in groups of two persons!

Problem 1 (6 points)

Let U be a random variable with uniform distribution on $[0,1]$ and $\{X_t : t \in [0,1]\}$ be the so-called waiting process associated with U, that is $X_t = \mathbb{1}_{t>U}$, $t \in [0,1]$. Describe all n-dimensional distributions of the process X.

Remark: To describe a distribution means to provide a formula for its probability density function (if the distribution is absolutely continuous) or its probability mass function (if the distribution is discrete).

Hint: You may wish to start with one and two-dimensional distributions.

Problem 2 (6 points)

n devices start to operate at time $t = 0$. They operate independently of each other for random periods of time and after that they shut down. The operating time of the i-th device is a random variable (taking only positive values) with distribution function F. Let X_t be the number of operating devices at the instant $t > 0$. Describe one- and two-dimensional distributions of the process $\{X_t : t > 0\}$.

Problem 3 (6 points)

Let $\xi_1, \xi_2, \ldots$ be independent random variables defined on some probability space $(\Omega, \mathcal{F}, \mathbb{P})$ and having geometric distribution with parameter $p \in (0,1)$:

$$\mathbb{P}[\xi_i = k] = p(1-p)^{k-1}, \quad k \in \mathbb{N}, \;\; i \in \mathbb{N}.$$

Let $S_n = \xi_1 + \ldots + \xi_n$ and

$$X_i = \begin{cases} 1, & \text{if there is } n \in \mathbb{N} \text{ such that } S_n = i, \\ 0, & \text{otherwise.} \end{cases}$$

Let also $Y_1, Y_2, \ldots$ be independent Bernoulli random variables with $\mathbb{P}[Y_i = 1] = p$, $\mathbb{P}[Y_i = 0] = 1-p$ defined on some other probability space $(\Omega', \mathcal{F}', \mathbb{P}')$. Show that the stochastic processes $\{X_i : i \in \mathbb{N}\}$ and $\{Y_i : i \in \mathbb{N}\}$ have the same finite-dimensional distributions.

Problem 4 (6 points)

Consider the probability space $(\Omega, \mathcal{F}, \mathbb{P})$, where $\Omega = [0,1]$, $\mathcal{F}$ is the Borel σ-algebra on $[0,1]$, and $\mathbb{P}$ is the Lebesgue measure on $[0,1]$. Prove that it is possible to construct a sequence of independent identically distributed random variables defined on *this* probability space which

(a) take the values 0 and 1 with probabilities 1/2.

(b) are uniformly distributed on [0,1].