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Winter term 2013/14
October 23, 2013

Stochastik II

Exercise Sheet 2

Due: October 30, 2013

Note: Please submit exercise sheets in groups of two persons!

Problem 1 (6 points)

Let $\{X_t : t \in [0,1]\}$ be a stochastic process with continuous sample paths defined on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$. Show that the following subsets of Ω are measurable:

(a) $\{\omega \in \Omega : \sup_{t\in[0,1]} X_t(\omega) > 1\}$.

(b) $\{\omega \in \Omega : \exists t \in [0,1] \text{ such that } X_t(\omega) = 0\}$.

(c) $\{\omega \in \Omega : \text{the function } t \mapsto X_t(\omega) \text{ is increasing}\}$.

Problem 2 (6 points)

Consider a stochastic process $\{X_t : t \in [0,1]\}$ consisting of *independent*, identically distributed random variables X_t such that X_t is Bernoulli with parameter $1/2$, for every $t \in [0,1]$. Show that this process is *not* stochastically continuous and not continuous in L^1.

Problem 3 (6 points)

Construct two stochastic processes $\{X_t : t \in [0,1]\}$ and $\{Y_t : t \in [0,1]\}$ whose one-dimensional distributions coincide but such that the two-dimensional distributions of these processes are different. (That is, the random variable X_t should have the same distribution as the random variable Y_t for every $t \in [0,1]$, but there should exist $t_1, t_2 \in [0,1]$ for which the distributions of the random vectors (X_{t_1}, X_{t_2}) and (Y_{t_1}, Y_{t_2}) are different).

Problem 4 (6 points)

Let $\mathbb{R}^{\mathbb{N}}$ be the set of all functions $f : \mathbb{N} \to \mathbb{R}$. Show that the following subsets of $\mathbb{R}^{\mathbb{N}}$ belong to the cylinder σ-algebra:

(a) The set of all functions $f : \mathbb{N} \to \mathbb{R}$ such that $\lim_{n\to\infty} f(n) = 0$.

(b) The set of all functions $f : \mathbb{N} \to \mathbb{R}$ which are increasing, that is $f(n+1) > f(n)$ for every n.

(c) The set of all functions $f : \mathbb{N} \to \mathbb{R}$ such that $\lim_{n\to\infty} f(n)$ exists.

Hint: If B is a Borel subset of $\mathbb{R}^n$ and $t_1, \dots, t_n \in \mathbb{N}$, then the set $\{f : \mathbb{N} \to \mathbb{R} : (f(t_1), \dots, f(t_n)) \in B\}$ is in the cylinder σ-algebra. You can use this without proof.