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Winter term 2013/14
October 29th, 2013

Stochastik II

Exercise Sheet 3

Due: November 5th, 2013

Note: Please submit exercise sheets in groups of two persons!

Problem 1 (6 points)

Suppose that weather can be either sunny (state 0) or rainy (state 1). If the weather is sunny on one day, then the next day will be sunny with probability $1-p$ and rainy with probability p. If the weather is rainy on one day, then the next day will be rainy with probability $1-q$ and sunny with probability q, where $p, q \in (0,1)$.

(a) Show that the n-step transition matrix of the corresponding Markov chain is equal to

$$P^n = \frac{1}{p+q}\begin{pmatrix} q & p \\ q & p \end{pmatrix} + \frac{(1-p-q)^n}{p+q}\begin{pmatrix} p & -p \\ -q & q \end{pmatrix}.$$

(b) Show that $\lim_{n\to\infty} P^n = \frac{1}{p+q}\begin{pmatrix} q & p \\ q & p \end{pmatrix}$.

Hint to (a): Use induction over n.

Problem 2 (6 points)

Three girls A, B and C are playing table tennis. In each game, two of the girls are playing against each other and the third girl does not play. In game $n+1$, the winner of game n plays against the girl which did not participated in game n. The probability that girl x beats girl y in any game is $s_x/(s_x+s_y)$, where $x, y \in \{A, B, C\}$, $x \neq y$, and $s_A, s_B, s_C > 0$ represent the "strengths" of the girls. Denote by X_n the girl which is *not* playing the n-th game.

(a) Construct the transition matrix of this Markov chain.

(b) Assume that in the first game, the girls A and B play. Determine the probability that the same girls will play each other again in the fourth game.

Problem 3 (6 points)

Let $X_0, X_1, \dots$ be independent identically distributed random variables with values $1, \dots, N$ and probabilities $\mathbb{P}[X_i = k] = a_k$, where $k = 1, \dots, N$.

(a) Show that $X_0, X_1, \dots$ is a Markov chain and compute its transition matrix and initial distribution.

(b) Compute the invariant probability measure of this Markov chain.