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Winter term 2013/14
November 6th, 2013

Stochastik II

Exercise Sheet 4

Due: November 13th, 2013

Note: Please submit exercise sheets in groups of two persons!

Problem 1 (6 points)

Consider a Markov chain with transition matrix

$$P = \begin{pmatrix} \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} \\ 0 & \frac{1}{4} & 0 & \frac{3}{4} & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} \end{pmatrix}.$$

(a) Identify the communicating classes of this Markov chain. Which of these classes are closed?

(b) Which states of this chain are recurrent?

Please provide an explanation, don't just write the answer.

Problem 2 (6 points)

A fair die (Würfel) is rolled repeatedly. At time n, denote by X_n the time since the most recent six (also, set $X_0 = 0$). For example, if the rolls are $5, 5, 6, 1, 3, 4, 6, 6, 2, 2, 5, \ldots$, then the sequence $X_1, X_2, \ldots$ is $1, 2, 0, 1, 2, 3, 0, 0, 1, 2, 3, \ldots$.

(a) Write down the transition matrix of this Markov chain.

(b) Show that this chain is irreducible and that every state is recurrent.

Problem 3 (6 points)

A house contains 5 rooms, A, B, C, D, E. Room C is connected to all other rooms. Also, there is a connection between room A and room B, as well as a connection between room D and room E. There are no further connections. A cat performs a random walk in the house in the following way. It starts in room A. After staying for 1 hour in a room, the cat goes to another room chosen at random from the set of rooms connected to the present location of the cat. All rooms connected to the present location of the cat have equal probabilities to be chosen. For example, if the cat is in B, it goes to A with probability $1/2$ or to C with probability $1/2$. Compute the unique invariant probability measure of this Markov chain.

Problem 4 (6 points)

Consider two boxes. At the beginning the first box contains k white balls and the second box contains k black balls. Every unit of time one draws one ball from the first box and independently one ball from the second box at random, all balls are equiprobable. Then, one places the ball drawn from the first box into the second box and the ball drawn from the second box into the first box. The procedure is repeated indefinitely. Let X_n be the number of white balls in the first box at time n. Write down the transition probabilities and find all invariant probability measures of this Markov chain.