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WS 2014/15
24.10.2014

Life Contingencies - Exercise Sheet 2

Presentation: Friday, 7th of November.

Exercise 1

How large must a half-yearly payment be in order that the stream of payments starting immediately be equivalent (in present value terms) at 6 % interest to a lump-sum payment of \$5000, if the payment-stream is to last

(a) 10 years,

(b) 20 years,

(c) forever?

Exercise 2

A survival model follows Makeham's law, so that

$$\mu_x = A + Bc^x \quad \text{for } x \geq 0.$$

(a) Show that under the Makeham's law

$$_tp_x = s^t g^{c^x(c^t-1)},$$

where $s = e^{-A}$ and $g = \exp(-B/\log c)$.

(b) Suppose you are given the values of $_{10}p_{50}$, $_{10}p_{60}$ and $_{10}p_{70}$. Show that

$$c = \left(\frac{\log({}_{10}p_{70}) - \log({}_{10}p_{60})}{\log({}_{10}p_{60}) - \log({}_{10}p_{50})}\right)^{0.1}.$$

Exercise 3

(a) Show that

$$\mathring{e}_x = \frac{1}{S_0(x)} \int_x^\infty S_0(t)dt,$$

where $S_0(t) = 1 - F_0(t)$, and hence, or otherwise, prove that

$$\frac{d}{dx}\mathring{e}_x = \mu_x \mathring{e}_x - 1.$$

Hint: $\frac{d}{dx}\{\int_a^x g(t)dt\} = g(x)$. What about $\frac{d}{dx}\{\int_x^a g(t)dt\}$?

(b) Deduce that

$$x + \mathring{e}_x$$

is an increasing function of x, and explain this result intuitively.

Exercise 4

Let $F_0(t) = 1 - e^{-\lambda t}$, where $\lambda > 0$.

(a) Show that $S_x(t) = e^{-\lambda t}$.

(b) Show that $\mu_x = \lambda$.

(c) Show that $e_x = (e^{\lambda} - 1)^{-1}$.

(d) What conclusions do you draw about using this lifetime distribution to model human mortality?