Exercise 1

Consider an endowment insurance with benefit payable at the end of the year of death.

(a) Show that the EPV is given by
\[
A_{x:m} = \sum_{k=0}^{n-2} v^{k+1} k q_x + v^n n - 1 p_x.
\]

(b) Compare this formula with
\[
A_{x:m} = \sum_{k=0}^{n-1} v^{k+1} k q_x + v^n P[K_x \geq n]
\]
and comment on the differences.

Exercise 2

Under an endowment insurance issued to a life aged $x$, let $X$ denote the present value of a unit sum insured, payable at the moment of death or at the end of the $n$-year term.

Under a term insurance issued to a life aged $x$, let $Y$ denote the present value of a unit sum insured, payable at the moment of death within the $n$-year term. Given that $V[X] = 0.0052$, $v^n = 0.3$, $n p_x = 0.8$, $E[Y] = 0.04$, calculate $V[Y]$.

Exercise 3

(a) Describe in words the insurance benefits with the present values given below.

(i) \[ Z_1 = \begin{cases} 20v^{T_x}, & \text{if } T_x \leq 15 \\ 10v^{T_x}, & \text{if } T_x > 15. \end{cases} \]

(ii) \[ Z_2 = \begin{cases} 0, & \text{if } T_x \leq 5 \\ 10v^{T_x}, & \text{if } 5 < T_x \leq 15. \\ 10v^{15}, & \text{if } T_x > 15. \end{cases} \]

(b) Write down in integral form the formulas for the expected value for $Z_1$ and $Z_2$.

(c) Derive expressions in terms of standard actuarial functions for the expected values of $Z_1$ and $Z_2$.

(d) Derive an expression in terms of standard actuarial functions for the covariance of $Z_1$ and $Z_2$. 
Exercise 4
A life insurance policy issued to a life aged 50 pays $2000 at the end of the quarter year of death before age 65 and $1000 at the end of the quarter year of death after age 65. Use the Standard Ultimate Survival Model \(^1\) with interest at 5% per year, in the following.

(a) Calculate the EPV of the benefit.

(b) Calculate the standard deviation of the present value of the benefit.

(c) The insurer charges a single premium of $500. Assuming that the insurer invests all funds at exactly 5% per year effective, what is the probability that the policy benefit has greater value than the accumulation of the single premium?

Exercise 5
Show that if \(\nu_y = -\log p_y \) for \(y = x, x+1, x+2, \ldots\), then under the assumption of a constant force of mortality between integer ages,

\[
\bar{A}_x = \sum_{t=0}^{\infty} v^t t p_x \frac{\nu_{x+t}(1 - v p_{x+t})}{\delta + \nu_{x+t}},
\]

where \(\delta = \log(1 + i)\) is the continuously compounded rate of interest and \(\bar{A}_x\) denotes the EPV of a whole life insurance in the continuous case.

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\(^1\)Makeham’s law with parameters \(A = 0.00022, B = 2.7 \cdot 10^{-6}\) and \(c = 1.124\).