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Life Contingencies - Exercise Sheet 3

Presentation: Friday, 21st of November.

Exercise 1

Consider an endowment insurance with benefit payable at the end of the year of death.

(a) Show that the EPV is given by

$$A_{x:\overline{n}|} = \sum_{k=0}^{n-2} v^{k+1} \; {}_{k|}q_x + v^n \; {}_{n-1}p_x.$$

(b) Compare this formula with

$$A_{x:\overline{n}|} = \sum_{k=0}^{n-1} v^{k+1} \; {}_{k|}q_x + v^n P[K_x \geq n]$$

and comment on the differences.

Exercise 2

Under an endowment insurance issued to a life aged x, let X denote the present value of a unit sum insured, payable at the moment of death or at the end of the n-year term.

Under a term insurance issued to a life aged x, let Y denote the present value of a unit sum insured, payable at the moment of death within the n-year term. Given that

$$\mathsf{V}[X] = 0.0052, \quad v^n = 0.3, \quad {}_np_x = 0.8, \quad \mathsf{E}[Y] = 0.04,$$

calculate $\mathsf{V}[Y]$.

Exercise 3

(a) Describe in words the insurance benefits with the present values given below.

$$\text{(i)} \quad Z_1 = \begin{cases} 20v^{T_x}, & \text{if } T_x \leq 15 \\ 10v^{T_x}, & \text{if } T_x > 15. \end{cases} \qquad \text{(ii)} \quad Z_2 = \begin{cases} 0, & \text{if } T_x \leq 5 \\ 10v^{T_x}, & \text{if } 5 < T_x \leq 15. \\ 10v^{15}, & \text{if } T_x > 15. \end{cases}$$

(b) Write down in integral form the formulas for the expected value for Z_1 and Z_2.

(c) Derive expressions in terms of standard actuarial functions for the expected values of Z_1 and Z_2.

(d) Derive an expression in terms of standard actuarial functions for the covariance of Z_1 and Z_2.

Exercise 4

A life insurance policy issued to a life aged 50 pays \$2000 at the end of the quarter year of death before age 65 and \$1000 at the end of the quarter year of death after age 65. Use the Standard Ultimate Survival Model [1] with interest at 5% per year, in the following.

(a) Calculate the EPV of the benefit.

(b) Calculate the standard deviation of the present value of the benefit.

(c) The insurer charges a single premium of \$500. Assuming that the insurer invests all funds at exactly 5% per year effective, what is the probability that the policy benefit has greater value than the accumulation of the single premium?

Exercise 5

Show that if $\nu_y = -\log\ p_y$ for $y = x, x+1, x+2, \ldots$, then under the assumption of a constant force of mortality between integer ages,

$$\bar{A}_x = \sum_{t=0}^{\infty} v^t\ {}_tp_x \frac{\nu_{x+t}(1 - vp_{x+t})}{\delta + \nu_{x+t}},$$

where $\delta = \log(1+i)$ is the continuously compounded rate of interest and $\bar{A}_x$ denotes the EPV of a whole life insurance in the continuous case.

[1] Makeham's law with parameters $A = 0.00022$, $B = 2.7 \cdot 10^{-6}$ and $c = 1.124$.