Dr. Akim Adekpedjou
Dipl.-Math. Stefan Roth

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Life Contingencies - Exercise Sheet 4

Presentation: Friday, 5th of December.

Exercise 1

(a) Describe the annuity with the following present value random variable:

$$Y = \begin{cases} v^{T_x}\bar{a}_{\overline{n-T_x}|} & \text{, if } T_x < n \\ 0 & \text{, if } T_x \geq n. \end{cases}$$

(b) Show that $\mathbb{E}[Y] = \bar{a}_{\overline{n}|} - \bar{a}_{x:\overline{n}|}$.

(c) Explain the answer in (b) by general reasoning.

Exercise 2

Consider the following portfolio of annuities-due currently being paid from the assets of a pension fund. Each annuity has an annual payment of \$ 10 000 as long as the annuitant survives. The lives are assumed

Age	Number of annuitants
60	40
70	30
80	10

to be independent. Calculate*

(a) the expected present value of the total outgo on annuities,

(b) the standard deviation of the present value of the total outgo on annuities, and

(c) the 95th percentile of the distribution of the present value of the total outgo on annuities using a normal approximation.

Exercise 3

Consider the random variables $Y = \bar{a}_{\overline{T_x}|}$ and $Z = v^{T_x}$.

(a) Derive an expression for the covariance, in terms of standard actuarial functions.

(b) Show that the covariance is negative.

(c) Explain this result in words.

*Use the standard ultimate model. Values can be found in the table that you will receive in the lesson next week.

Exercise 4

Let $H = \min\{K_x, n\}$.

(a) Show that

$$\mathsf{Var}[a_{\overline{H}|}] = \frac{{}^2A_{x:\overline{n+1}|} - (A_{x:\overline{n+1}|})^2}{d^2}$$

(b) Prove, that

$$\frac{(1+i)^2[A^1_{x:\overline{n}|} - (A^1_{x:\overline{n}|})^2] - 2(1+i)A^1_{x:\overline{n}|}v^n\ {}_np_x + v^{2n}\ {}_np_x(1 - {}_np_x)}{i^2}$$

is equal to the expression in (a).