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Life Contingencies - Exercise Sheet 5

Presentation: Friday, 9th and Tuesday, the 13th of January.

Exercise 1

Let

$$S_0(x) = \exp\left\{-\left(Ax + \frac{1}{2}Bx^2 + \frac{C}{\log D}D^x - \frac{C}{\log D}\right)\right\}$$

where A, B, C and D are all positive.

(a) Show that the function S_0 is a survival function.

(b) Derive a formula for $S_x(t)$.

(c) Derive a formula for μ_x.

Exercise 2

Given

$$F_0(x) = 1 - \frac{1}{1+x}, \quad \text{for } x \geq 0,$$

find expressions for (a), (b), (c) below, simplifying as far as possible,

(a) $S_0(x)$,

(b) $f_0(x)$,

(c) $S_x(t)$,

and calculate:

(d) p_{20}, and

(d) ${}_{10|5}q_{30}$.

Exercise 3

Show that

$$\frac{d}{dx}{}_tp_x = {}_tp_x(\mu_x - \mu_{x+t}).$$

Exercise 4

Show that for integer n,

$$e_{x:\overline{n}|} = \sum_{k=1}^{n} {}_kp_x.$$

Exercise 5

(a) Show that
$$\mathring{e}_x \leq \mathring{e}_{x+1} + 1$$

(b) Show that
$$\mathring{e}_x \geq e_x$$

(c) Explain (in words) why
$$\mathring{e}_x \approx e_x + \frac{1}{2}$$

Exercise 6

You are given the following table of values for l_x and A_x, assuming an effective interest rate of 6% per year.

x	l_x	A_x
35	100000.00	0.151375
36	99737.15	0.158245
37	99455.91	0.165386
38	99154.72	0.172804
39	98831.91	0.180505
40	98485.68	0.188492

Calculate

(a) ${}_5E_{35}$,

(b) $A^{1}_{35:\overline{5}|}$

(c) ${}_{5|}A_{35}$.

Exercise 7

Calculate A_{70} given that

$$A_{50:\overline{20}|} = 0.42247, \quad A^{1}_{50:\overline{20}|} = 0.14996, \quad A_{50} = 0.31266.$$

Exercise 8

Use Jensen's inequality to show that
$$\bar{a}_x \leq \bar{a}_{\overline{\mathbb{E}[T_x]}|}.$$

Exercise 9

Find, and simplify where possible:

(a) $\frac{d}{dx}\ddot{a}_x$ and

(b) $\frac{d}{dx}\ddot{a}_{x:\overline{n}|}$.

Exercise 10

The force of mortality for a certain population is exactly half the sum of the forces of mortality in two standard mortality tables, denoted A and B. Thus

$$\mu_x = (\mu_x^A + \mu_x^B)/2$$

for all x. A student has suggested the approximation

$$a_x = (a_x^A + a_x^B)/2.$$

Will this approximation overstate or understate the true value of a_x?