Exercise 1
Let
\[ S_0(x) = \exp \left\{ - \left( A x + \frac{1}{2} B x^2 + \frac{C}{\log D} D^x - \frac{C}{\log D} \right) \right\} \]
where \( A, B, C \) and \( D \) are all positive.

(a) Show that the function \( S_0 \) is a survival function.

(b) Derive a formula for \( S_x(t) \).

(c) Derive a formula for \( \mu_x \).

Exercise 2
Given
\[ F_0(x) = 1 - \frac{1}{1+x}, \quad \text{for } x \geq 0, \]
find expressions for (a), (b), (c) below, simplifying as far as possible,

(a) \( S_0(x) \),

(b) \( f_0(x) \),

(c) \( S_x(t) \),

and calculate:

(d) \( p_{20} \), and

(d) \( 10|s_{30} \).

Exercise 3
Show that
\[ \frac{d}{dx} t p_x = t p_x (\mu_x - \mu_{x+t}). \]

Exercise 4
Show that for integer \( n \),
\[ e_{x:n} = \sum_{k=1}^{n} k p_x. \]
Exercise 5

(a) Show that
\[ \bar{e}_x \leq \bar{e}_{x+1} + 1 \]

(b) Show that
\[ \bar{e}_x \geq e_x \]

(c) Explain (in words) why
\[ \bar{e}_x \approx e_x + \frac{1}{2} \]

Exercise 6

You are given the following table of values for \( l_x \) and \( A_x \), assuming an effective interest rate of 6% per year.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( l_x )</th>
<th>( A_x )</th>
</tr>
</thead>
<tbody>
<tr>
<td>35</td>
<td>100000.00</td>
<td>0.151375</td>
</tr>
<tr>
<td>36</td>
<td>99737.15</td>
<td>0.158245</td>
</tr>
<tr>
<td>37</td>
<td>99455.91</td>
<td>0.165386</td>
</tr>
<tr>
<td>38</td>
<td>99154.72</td>
<td>0.172804</td>
</tr>
<tr>
<td>39</td>
<td>98831.91</td>
<td>0.180505</td>
</tr>
<tr>
<td>40</td>
<td>98485.68</td>
<td>0.188492</td>
</tr>
</tbody>
</table>

Calculate

(a) \( 5E_{35} \),
(b) \( A_{35:51} \)
(c) \( 5|A_{35} \).

Exercise 7

Calculate \( A_{70} \) given that
\[ A_{50:20} = 0.42247, \quad A_{50} = 0.14996, \quad A_{50} = 0.31266. \]

Exercise 8

Use Jensen’s inequality to show that
\[ \bar{a}_x \leq \bar{a}_{E[T_x]} \]

Exercise 9

Find, and simplify where possible:

(a) \( \frac{d}{dx} \bar{a}_x \) and
(b) \( \frac{d}{dx} \bar{a}_{x|m} \)
Exercise 10
The force of mortality for a certain population is exactly half the sum of the forces of mortality in two standard mortality tables, denoted A and B. Thus

$$\mu_x = (\mu_x^A + \mu_x^B)/2$$

for all $x$. A student has suggested the approximation

$$a_x = (a_x^A + a_x^B)/2.$$ 

Will this approximation overstate or understate the true value of $a_x$?