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winter term 2014/2015
October 17, 2014

Statistical methods of risk theory

Exercise Sheet 1

Due to: October 24, 2014

Note: Solutions may be submitted in groups of up to three students!

Problem 1 (6 points)

Show that the following families of distributions belong to the exponential dispersion families

(a) $\{\text{Exp}(\lambda),\ \lambda > 0\}$,

(b) $\{\text{Geo}(p),\ p \in (0,1)\}$,

(c) $\{\text{Gamma}(\alpha, \beta),\ \alpha, \beta > 0\}$.

Problem 2 (6 points)

Compute the moment generating function $\hat{m}_X$ of a random variable X in the case that

(a) $X \sim \text{Exp}(\lambda),\ \lambda > 0$,

(b) $X \sim \text{Geo}(p),\ p \in (0,1)$,

(c) $X \sim \text{Gamma}(\alpha, \beta),\ \alpha, \beta > 0$.

Problem 3 (6 points)

Consider the Geo-Beta-Model. That means that for a given value of the parameter $p \in (0,1)$ the random variables $X_1, \ldots, X_n$ are independent and identically geometric distributed with parameter p. The a-priori distribution for p is the Beta-distribution on $(0,1)$ with deterministic and known parameters $\alpha, \beta > 0$. The realisation $(x_1, \ldots, x_n)$ of $(X_1, \ldots, X_n)$ is considered. Determine the a-posteriori distribution of p and the Bayes estimator $\hat{p}$ for p.