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Statistical methods of risk theory

Exercise Sheet 2

Due to: October 31, 2014

Note: Solutions may be submitted in groups of up to three students!

Problem 1 (6 points)

Consider an exponential dispersion family $EDF(\Theta, C, b, a)$ with known dispersion parameter $\phi > 0$ and unknown natural parameter $\theta \in (\theta_{\min}, \theta_{\max})$ with $-\infty \leqslant \theta_{\min} < \theta_{\max} \leqslant \infty$ and assume that C does not consist of a single point. Assume that one realisation $x_1, \ldots, x_n$ of i.i.d. random variables $X_1, \ldots, X_n$ is observed and that

$$b'(\theta) = \bar{x}_n \tag{1}$$

has a solution. Show that the solution of (1) is uniquely determined and show that the maximum-likelihood-estimator $\hat{\theta}$ of θ exists and is a solution of (1).
Hint: Consider $b''(\theta)$ in order to show the uniqueness.

Problem 2 (6 points)

Let $X_1, \ldots, X_n$ be i.i.d. random variables where $X_i \sim \Gamma(\alpha, \beta)$, $\alpha > 0$ is known and $\beta > 0$ is unknown. The realisation $(x_1, \ldots, x_n)$ of $(X_1, \ldots, X_n)$ is considered. Use conjugate priors in order to determine the a-priori distribution as well as the a-posteriori distribution of $\widetilde{\beta}$ and the Bayes estimator of $\mathbb{E}X_1$.

Problem 3 (6 points)

Let $X_1, \ldots, X_m$ be i.i.d. random variables where $X_i \sim Bin(n, p)$ and $p \in (0, 1)$ is unknown. The realisation $(x_1, \ldots, x_m)$ of $(X_1, \ldots, X_m)$ is considered. Use conjugate priors in order to determine the a-priori distribution as well as the a-posteriori distribution of $\widetilde{p}$ and the Bayes estimator of $\mathbb{E}X_1$.
Hint: You have to use a distinct exponential dispersion family for each integer n, i.e. you have to allow $a(x, \phi)$ and $b(\theta)$ to depend on n.