Let \( X_1, \ldots, X_n : \Omega \to \mathbb{N}_0 \) be a collection of random variables, such that \( \mathbb{P}(X_1 = k_1, \ldots, X_n = k_n) > 0 \) for all \( k_1, \ldots, k_n \in \mathbb{N}_0 \). Then, the joint distribution of \( X_1, \ldots, X_n \) is uniquely determined by the family of probabilities

\[
\{ \mathbb{P}(X_1 = k_1, \ldots, X_n = k_n), k_1, \ldots, k_n \geq 1 \}. \tag{1}
\]

This holds because for arbitrary \( m \) with \( 1 \leq m < n \)

\[
\begin{align*}
\mathbb{P}(X_1 = 0, \ldots, X_m = 0, X_{m+1} = k_{m+1}, \ldots, X_n = k_n) & = \mathbb{P}(X_1 = 0, \ldots, X_m = 0 \mid X_{m+1} = k_{m+1}, \ldots, X_n = k_n) \mathbb{P}(X_{m+1} = k_{m+1}, \ldots, X_n = k_n) \\
& = \left( 1 - \sum_{k_1 + \ldots + k_m \geq 1} \left( \mathbb{P}(X_1 = k_1, \ldots, X_m = k_m \mid X_{m+1} = k_{m+1}, \ldots, X_n = k_n) \right) \right) \\
& \quad \mathbb{P}(X_{m+1} = k_{m+1}, \ldots, X_n = k_n) \\
& = \mathbb{P}(X_{m+1} = k_{m+1}, \ldots, X_n = k_n) - \sum_{k_1 + \ldots + k_m \geq 1} \mathbb{P}(X_1 = k_1, \ldots, X_n = k_n)
\end{align*}
\]

Then we have reduced it to the case in which \( m - 1 \) random variables are zero. By iteration we reduce the computation of

\[
\mathbb{P}(X_1 = 0, \ldots, X_m = 0, X_{m+1} = k_{m+1}, \ldots, X_n = k_n)
\]

to computing probabilities of the form in (1). Note that if \( m = n \) we can write

\[
\mathbb{P}(X_1 = 0, \ldots, X_n = 0) = 1 - \sum_{k_1 + \ldots + k_n \geq 1} \mathbb{P}(X_1 = k_1, \ldots, X_n = k_n).
\]