Let $X_1, \ldots, X_n : \Omega \to \mathbb{N}_0$ be a collection of random variables, such that $\mathbb{P}(X_1 = k_1, \ldots, X_n = k_n) > 0$ for all $k_1, \ldots, k_n \in \mathbb{N}_0$. Then, the joint distribution of $X_1, \ldots, X_n$ is uniquely determined by the family of probabilities

$$\{\mathbb{P}(X_1 = k_1, \ldots, X_n = k_n), k_1, \ldots, k_n \geq 1\}. \tag{1}$$

This holds because for arbitrary m with $1 \leq m < n$

$$\begin{aligned}
&\mathbb{P}(X_1 = 0, \ldots, X_m = 0, X_{m+1} = k_{m+1}, \ldots, X_n = k_n) \\
&\quad = \mathbb{P}(X_1 = 0, \ldots, X_m = 0 \mid X_{m+1} = k_{m+1}, \ldots, X_n = k_n)\mathbb{P}(X_{m+1} = k_{m+1}, \ldots, X_n = k_n) \\
&\quad = \left(1 - \sum_{k_1+\ldots+k_m \geq 1} (\mathbb{P}(X_1 = k_1, \ldots, X_m = k_m \mid X_{m+1} = k_{m+1}, \ldots, X_n = k_n))\right) \\
&\quad \mathbb{P}(X_{m+1} = k_{m+1}, \ldots, X_n = k_n) \\
&\quad = \mathbb{P}(X_{m+1} = k_{m+1}, \ldots, X_n = k_n) - \sum_{k_1+\ldots+k_m \geq 1} \mathbb{P}(X_1 = k_1, \ldots X_n = k_n)
\end{aligned}$$

Then we have reduced it to the case in which $m-1$ random variables are zero. By iteration we reduce the computation of

$$\mathbb{P}(X_1 = 0, \ldots, X_m = 0, X_{m+1} = k_{m+1}, \ldots, X_n = k_n)$$

to computing probabilities of the form in (1). Note that if $m = n$ we can write

$$\mathbb{P}(X_1 = 0, \ldots, X_n = 0) = 1 - \sum_{k_1+\ldots+k_m \geq 1} \mathbb{P}(X_1 = k_1, \ldots X_n = k_n).$$