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Prof. Dr. Volker Schmidt
Matthias Neumann

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Stochastics II
Exercise Sheet 4

Deadline: November 12, 2014 at 4pm before the exercises

Exercise 1 (2.5 + 2.5)

Let $\{N_t, t \geq 0\}$ be a Cox process with random intensity function $\lambda_t = \sin(t + U)$, where $U : \Omega \to [0, \infty)$ is an arbitrary non-negative random variable.

a) Let $U = 0$ a.s. Show that the process $\{N_t, t \geq 0\}$ has independent increments, but does not have stationary increments.

b) Let $U \sim U(0, 2\pi)$. Show that the process $\{N_t, t \geq 0\}$ has stationary increments, but does not have independent increments.

Exercise 2 (5)

Let $\{X_t, t \geq 0\}$ be a compound Poisson process, i.e. $X_t = \sum_{i=1}^{N_t} U_i$, where $U_1, U_2, \ldots$ are i.i.d. and $\{N_t, t \geq 0\}$ is a Poisson process with intensity $\lambda > 0$. Let $U_1 \sim \text{Exp}(\gamma)$ with $\gamma > 0$. Compute $\mathbb{E}X_t^3$ for each $t \geq 0$.

Exercise 3 (5)

Let $\{N_t, t \geq 0\}$ be a Cox process with random intensity function $\lambda_t = Z$, where $\mathbb{P}(Z = \lambda_1) = \mathbb{P}(Z = \lambda_2) = 1/2$ for some $\lambda_1, \lambda_2 > 0$ with $\lambda_1 \neq \lambda_2$. Compute the moment generating function, expectation and variance of N_t for each $t > 0$.

Exercise 4 (5)

Let $\{X_t, t \geq 0\}$ be a Markov process with finite state space and transition function $\{\mathbf{P}(h), h \geq 0\}$. Show that $\mathbf{P}(h)$ is uniformly continuous in $h \geq 0$.