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Stochastics II
Exercise Sheet 5

Deadline: November 19, 2014 at 4pm before the exercises

Exercise 1 (4)

Let $\{X_t, t \geq 0\}$ be a Markov process with finite state space and transition function $\{\mathbf{P}(h), h \geq 0\}$. Show that $p_{ii}(h) > 0$ for all $i \in E$ and all $h \geq 0$.

Exercise 2 (3+3)

Let $\{X_t, t \geq 0\}$ be a $\mathbb{N}_0$-valued stochastic process with independent and stationary increments. Furthermore let the trajectories of $\{X_t, t \geq 0\}$ be functions which are continuous almost everywhere (w.r.t. the Lebesgue-measure) with probability 1.

a) Show that $\{X_t, t \geq 0\}$ is a Markov process.

b) Let $\{N_t, t \geq 0\}$ be a homogeneous Poisson process with intensity $\lambda > 0$. Conclude that $\{N_t, t \geq 0\}$ is a Markov process and compute the transition function $\{\mathbf{P}(h), h \geq 0\}$ as well as the entries q_{ij} of the intensity matrix $Q = (q_{ij})_{i,j \in \mathbb{N}_0}$ for all $i, j \in \mathbb{N}_0$.

Exercise 3 (5)

Let $\{X_t, t \geq 0\}$ be a Markov process with finite state space $E = \{1, \ldots, \ell\}$, transition function $\{\mathbf{P}(h), h \geq 0\}$ and intensity matrix Q. Let $\theta_1, \ldots, \theta_\ell$ be the eigenvalues of Q, such that $\theta_i \neq \theta_j$ for all $i \neq j \in E$. Let $\phi_1, \ldots, \phi_\ell$ be the right and $\psi_1, \ldots, \psi_\ell$ be the left eigenvectors of Q. Show that $\mathbf{P}(h) = \sum_{i=1}^{\ell} \exp(h\theta_i)\phi_i\psi_i^\top$ for each $h > 0$.

Exercise 4 (3+3)

Let $\{X_t, t \geq 0\}$ be a Markov process with state space $E = \{1, 2\}$ and intensity matrix

$$Q = \begin{pmatrix} -\mu & \mu \\ \lambda & -\lambda \end{pmatrix}$$

for some $\lambda, \mu > 0$.

a) Compute $p_{ij}(h)$ for all $i, j \in E$ and $h > 0$ by means of the Kolmogorov forward equation.

b) Compute $\mathbf{P}(h)$ by the aid of Exercise 3.