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# Stochastics II
# Exercise Sheet 7

Deadline: December 3, 2014 at 4pm before the exercises

**Exercise 1** (4)

Let $\{X_t, t \geq 0\}$ and $\{Y_t, t \geq 0\}$ be two independent Wiener processes defined on the same probability space $(\Omega, \mathcal{F}, \mathbb{P})$. Let $a, b \in \mathbb{R}$. Define the stochastic process $\{Z_t, t \geq 0\}$ by $Z_t = aX_t + bY_t$ for each $t \geq 0$. For which constellations of $a, b$ is $\{Z_t, t \geq 0\}$ a Wiener process?

**Exercise 2** (2+3)

Let $\{X_t, t \in [0,1]\}$ be a Wiener process and define the stochastic process $\{B_t, t \in [0,1]\}$ by $B_t = X_t - t \cdot X_1$ for each $t \in [0,1]$.

a) Compute $\operatorname{Cov}(B_s, B_t)$ for arbitrary $s, t \in [0,1]$.

b) Compute $\operatorname{Cov}(B_{s_1} - B_{s_2}, B_{t_1} - B_{t_2})$ for arbitrary $0 \leq t_2 \leq t_1 \leq s_2 \leq s_1 \leq 1$ and show that $\{B_t, t \in [0,1]\}$ does not have independent increments.

**Exercise 3** (2+3+3+2)

Let $s \geq 0$ be arbitrary and let $\{X_t, t \geq 0\}$ be a Wiener process. Define the sequence of random variables $Z_1, Z_2, \ldots$ by

$$Z_n = -s + \sum_{i=1}^{2^n} (X_{is/2^n} - X_{(i-1)s/2^n})^2,$$

for each $n \in \mathbb{N}$. Show:

a) $\mathbb{E} Z_n = 0$, for each $n \in \mathbb{N}$

b) $\operatorname{Var} Z_n = s^2 2^{1-n}$ for each $n \in \mathbb{N}$

c) $\lim_{n\to\infty} Z_n = 0$ a.s.

d) $\lim_{n\to\infty} \sum_{i=1}^{2^n} \left| X_{is/2^n} - X_{(i-1)s/2^n} \right| = \infty$ a.s.

**Exercise 4** (5)

Write an R- or Matlab program in order to simulate an approximation of the Wiener process on $[0, 2]$. Use the first 10 Schauder functions for the approximation. Hand in your code and a plot of one realization.