Prof. Dr. Volker Schmidt
Matthias Neumann

Winter term 2014/15

Stochastics II
Exercise Sheet 9

Deadline: December 17, 2014 at 4pm before the exercises

Exercise 1 (6)

Let $\{X_t, t \geq 0\}$ be a Wiener process on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$ and let $c, t_0 > 0$ be arbitrary. Define the stochastic processes $\{Y_t^{(i)}, t \geq 0\}$ for $i \in \{1,2,3\}$ by $Y_t^{(1)} = -X_t$, $Y_t^{(2)} = X_{t+t_0} - X_{t_0}$ and $Y_t^{(3)} = \sqrt{c} X_{t/c}$ for each $t \geq 0$. Show that $\{Y_t^{(i)}, t \geq 0\}$ is a Wiener process on $(\Omega, \mathcal{F}, \mathbb{P})$ for each $i \in \{1,2,3\}$.

Exercise 2 (3+2+2+2)

Let $\{X_t, t \geq 0\}$ be a Wiener process. Define $M_t = \max_{0 \leq s \leq t} X_s$ for each $t \geq 0$ and define $T_a = \inf\{t \geq 0 : X_t = a\}$.

a) Show that $\mathbb{P}(X_t \leq a - y, M_t \geq a) = \mathbb{P}(X_t > a + y)$ for all $a > 0, y \geq 0$.

b) Show that $\mathbb{P}(M_t \geq a) = 2\mathbb{P}(X_t \geq a)$ for each $a \geq 0$.

c) Determine the distribution of T_a for each $a > 0$.

d) Determine the distribution of M_t for each $t > 0$.

Hint: Define the process $\{X_t^\star, t \geq 0\}$ *by*

$$X_t^\star = X_t \mathbb{I}(t \leq T_a) + (2a - X_t)\mathbb{I}(t > T_a)$$

for each $t \geq 0$. *You may assume without proof that* $\{X_t^\star, t \geq 0\}$ *is a Wiener process.*

Exercise 3 (4)

Let $\{X_t, t \geq 0\}$ be a Wiener process and define $M_t = \max_{0 \leq s \leq t} X_s$. Show that the set $A_t = \{s \in [0,t] : X_s = M_t\}$ consists almost surely of one single point for each $t > 0$.