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Stochastics III
Extra Problem Sheet
(Reading Course Material)

Regarding all exercises: **Use R only as a calculator, not for doing all the work!**

Exercise 1

In a biological study the weight of 20 rats has been determined:

356.4	362.5	394.7	356.0	387.6
305.1	385.1	383.2	346.6	314.2
394.8	370.8	434.2	365.2	377.1
365.9	384.4	297.4	404.3	412.0

You can download the data in the file *rats.txt* from the course website.

a) Suppose it is known that the weight of this type of rat is usually normally distributed with mean $\mu_0 = 370$ grams and standard deviation $\sigma_0 = 27$ grams. Conduct a test for the hypothesis that the weight of the animals in the sample above is $\mathrm{N}(\mu_0, \sigma_0^2)$-distributed, using the Kolmogorov-Smirnov test with significance level $\alpha = 0.05$.

b) Plot the empirical distribution function of the sample above together with the distribution function of the normal distribution from a).

Hint: It is $s_{20,0.95} = 1.315$.

Exercise 2

The following numbers of calls have been registered in a hotel center in 50 time intervals of 15 minutes (each):

Number k of calls	0	1	2	3	4	5	6	7	8	9	10	>10
Number of time intervals with k calls	1	6	8	10	6	6	7	1	1	2	2	0

Use a χ^2 test to test whether the number of calls within 15 minutes is Poisson-distributed with parameter $\lambda = 3$ with a significance level of $\alpha = 0.01$. Choose the partition such that $np_{0,j} \geq 8$ for each class $j = 1, \ldots, r$.

Exercise 3

Let $X_1, \ldots, X_n$ be a sequence of independent and identically $\mathrm{Bin}(1, p)$-distributed random variables with $p \in (0, 1)$.

a) Show that the distribution of $D_n = \sup_{t\in\mathbb{R}} \left|\widehat{F}(t; x_1, \ldots, x_n) - F_0(t)\right|$ depends on p for every $n \in \mathbb{N}$.

b) Show that the asymptotic distribution of $\sqrt{n}D_n$ depends on p as well.

Exercise 4

The following data, which can be downloaded in the file *steel.txt* from the course website, contains the tensile strength of 30 steel panels (in kg/mm^2):

44.0	42.8	40.8	41.4	44.4	43.9	42.8	44.0	42.2	44.8
43.3	42.5	43.5	44.7	45.8	42.0	45.2	41.1	43.8	43.8
42.9	43.7	45.8	41.4	42.6	45.0	44.5	41.6	44.3	43.5

a) Use Pearson's χ^2 test to verify whether the tensile strength is normally distributed with mean 45 and variance 4 with significance level $\alpha = 0.05$. Partition the data into 4 classes such that $p_{0,1} = p_{0,2} = p_{0,3} = p_{0,4} = 0.25$.

b) Test whether the data follow a normal distribution (with unspecified parameters) using the χ^2 test of Pearson-Fisher and a significance level of $\alpha = 0.05$. Partition the data into classes of 6 elements.