Dr. Tim Brereton
Lisa Handl

Winter Term 2015/16

Methods of Monte Carlo Simulation

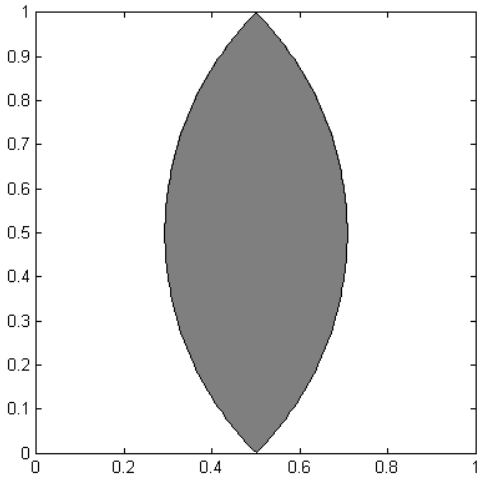
Problem Sheet 1

Deadline: October 29, 2015 at 4 pm before the exercises

Please email your code to lisa.handl@uni-ulm.de AND hand in a printed copy of the code!

Exercise 1 (programming) (3 points)

The gray region in the following plot is the intersection of two circles with centers $(0, \frac{1}{2})$ and $(1, \frac{1}{2})$. Calculate the radii of these circles and write a Matlab program to estimate the surface area of the gray region by throwing 10^2, 10^3 and 10^5 points randomly onto the unit square.



Exercise 2 (programming) (3 points)

Write a Matlab program which generates $N = 10^4$ pseudo-random numbers $\widetilde{U}_1, \ldots, \widetilde{U}_N$ using the uniform random generator RANDU. This is a linear congruential generator with parameters $a = 2^{16} + 3$, $c = 0$ and $m = 2^{31}$. Produce the following plots:

a) the sequence of pseudo-random numbers itself,

b) the points $\left(\widetilde{U}_i, \widetilde{U}_{i+1}\right)$ in a 2D scatter plot,

c) the points $\left(\widetilde{U}_i, \widetilde{U}_{i+1}, \widetilde{U}_{i+2}\right)$ in a 3D scatter plot.

Can you see a problem with any of the plots?

Hint: You might need the Matlab statements `plot(x)`, `scatter(x, y)` and `scatter3(x, y, z)`.

Note: RANDU was used by IBM in the 60s and 70s.

Exercise 3 (programming) (4 points)

Implement the random generator MRG32k3a and use it to approximately evaluate the integral

$$\iiint_{[0,1]^3} \cos(x_1 x_2) \cdot \sin(\cos(x_3 x_1)) \, \mathrm{d}x_1 \, \mathrm{d}x_2 \, \mathrm{d}x_3$$

based on $N = 10^4$ random points in $(0,1)^3$.

Please register at SLC for this lecture: `http://slc.mathematik.uni-ulm.de`
This is required to receive points for your problem sheets.