Dr. Tim Brereton
Lisa Handl

Winter Term 2015/16

Methods of Monte Carlo Simulation

Problem Sheet 2

Deadline: November 12, 2015 at 4 pm before the exercises

Please email your code to lisa.handl@uni-ulm.de AND hand in a printed copy of the code!

Exercise 1 (theory) (1 + 2 points)

A discounter sells low quality light bulbs. There is a 25 % chance that a light bulb fails immediately when it is first switched on. If it doesn't fail immediately, the lifetime of a light bulb (in years) is exponentially distributed with parameter $\lambda = 3$. Let X be the random variable describing the lifetime of such a light bulb.

a) Sketch the cumulative distribution function of X.

b) Write down a formula for how a standard uniformly distributed random variable can be transformed into X using the inverse-transform method.

Exercise 2 (programming) (2 points)

Write a Matlab program to generate 1000 pseudo-random numbers with the distribution of X from Exercise 1. Plot the empirical cumulative distribution function of the sample you obtain.

Hint: To plot the empirical cumulative distribution function of a vector in Matlab, use the function `cdfplot`.

Exercise 3 (programming) (3 points)

Consider the improper integral

$$\int_0^\infty 2x\mathrm{e}^{-2x}\,\mathrm{d}x.$$

Write a Matlab program to estimate the value of this integral using Monte Carlo integration with exponentially distributed pseudo-random numbers. Consider the parameters $\lambda = \frac{1}{2}$, 1 and 2. For each of them, generate 10^4 pseudo-random numbers with distribution $\mathrm{Exp}(\lambda)$ via the inverse-transform method and use them for estimation. Estimate the standard deviation of your estimator.

Exercise 4 **(theory)** (2 + 1 points)

a) Calculate the expectation and variance of the estimator you used in Exercise 3 a) as functions of λ, where $\lambda \in (0, \infty)$.

b) Determine the parameter $\lambda \in (0, \infty)$ which minimizes the variance of this estimator.

Exercise 5 **(theory)** (3 + 2 + 1 + 1 points)

Suppose you want to draw from the positive normal distribution, i.e., from the density

$$f(x) = \sqrt{\frac{2}{\pi}} \exp\left(-\frac{x^2}{2}\right) \mathbb{I}\{x \geq 0\},$$

using acceptance rejection. As proposals, you want to use exponentially distributed random numbers with some parameter $\lambda \in (0, \infty)$.

a) Calculate the optimal constant C for acceptance rejection as a function of λ.

b) What is the best parameter $\lambda \in (0, \infty)$ you could use for the proposals?

c) Using the optimal λ, how many of the generated exponentially distributed proposals do you expect to accept (as a percentage)?

d) Explain shortly how you could modify the algorithm to sample from a full (unconditional) normal distribution.