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WS 2015/2016

Stochastics II
Exercise Sheet 3

Due to: Wednesday, 4th of November 2015

Exercise 1 (6 Points)

Let $W = \{W(t);\ t \geq 0\}$ be the Wiener process. Which of the following processes are Wiener processes as well?

(a) $W_1 = \{W_1(t) = -W(t);\ t \geq 0\}$,

(b) $W_2 = \{W_2(t) = \sqrt{t}W(1);\ t \geq 0\}$,

(c) $W_3 = \{W_3(t) = W(2t) - W(t);\ t \geq 0\}$.

Exercise 2 (4 Points)

Give examples for a stochastic process $X = \{X(t);\ t \in T\}$ with the following properties (with proof!):

(a) X has L^2-differentiable paths which are not a.s. differentiable.

(b) X has a.s. differentiable paths which are not L^1-differentiable.

Exercise 3 (8 Points)

Let $\{T_n\}_{n\in\mathbb{N}}$ be a sequence of i.i.d. random variables with $T_1 \sim Exp(\lambda)$, $\lambda > 0$. The process $N = \{N(t);\ t \geq 0\}$ given by

$$N(t) = \sum_{n=1}^{\infty} \mathbb{1}_{\{T_1+\ldots,T_n \leq t\}}$$

is called a Poisson process with intensity λ.

(a) Prove: $N(t)$ is Poisson distributed for each $t > 0$.

(b) Determine the parameter of this Poisson distribution.

(c) Calculate $H(t) = \mathbb{E}N(t)$ (the so called renewal function).

(d) Show that the process N posesses independent increments.

Exercise 4 (4 Points)

Let $X = \{X(t);\ t \geq 0\}$ be a real valued stochastic process with independent increments. Show that X has stationary increments if the distribution of $X(t + h) - X(h)$ does not depend on h for arbitrary $t \geq 0$.