Stochastics II | **Universität Ulm**
SoSe 2016 | Prof. Dr. Evgeny Spodarev
October 18, 2016 | Dr. Vitalii Makogin

Exercise sheet 1 (total – 19 points) till October 26, 2016

Exercise 1-1 (3 points)

Show the existence of a random function with finite dimensional multivariate Gaussian distributions and specify spaces $(S_{t_1,\dots,t_n}, \mathcal{B}_{t_1,\dots,t_n})$.

Exercise 1-2 (6 points)

Let η be a random variable with distribution function F. Prove that $X(t), t \in \mathbb{R}$ is a stochastic process, if

1. $X(t) = \eta t, t \in \mathbb{R}$,
2. $X(t) = \min(\eta, t), t \in \mathbb{R}$,
3. $X(t) = \operatorname{sign}(\eta + t), t \in \mathbb{R}$.

Draw the sample paths of the process X. Find one-dimensional distributions of the process X.

Exercise 1-3 (3 points)

Two devices start to operate at the instant of time $t = 0$. They operate independently of each other for random periods of time and after that they shut down. The operating time of the $i-$th device has a distribution function $F_i, i = 1, 2$. Let $X(t)$ be the number of operating devices at the instant t. Find one- and two-dimensional distributions of the process $\{X(t), t \in \mathbb{R}_+\}$.

Exercise 1-4 (3 points)

Construct two stochastic processes $\{X_t, t \in [0, 1]\}$ and $\{Y_t, t \in [0, 1]\}$ whose one-dimensional distributions coincide but such that the two-dimensional distributions of these processes are different. (That is, the random variable X_t should have the same distribution as the random variable Y_t for every $t \in [0, 1]$, but there should exist $t_1, t_2 \in [0, 1]$ for which the distributions of the random vectors (X_{t_1}, X_{t_2}) and (Y_{t_1}, Y_{t_2}) are different).

Exercise 1-5 (4 points)

Consider the probability space $(\Omega, \mathcal{F}, \mathbf{P})$, where $\Omega = [0, 1]$, $\mathcal{F}$ is the Borel σ-algebra on $[0, 1]$, and $\mathbf{P}$ is the Lebesgue measure on $[0, 1]$. Prove that it is possible to construct a sequence of independent identically distributed random variables defined on this probability space which
(a) take the values 0 and 1 with probabilities 1/2,
(b) are uniformly distributed on [0,1].
Please, provide an explicit construction (not using theorems 1.1.1-2 from the Lecture notes).