Stochastics II — WS 2016/2017 — December 20, 2016

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Exercise sheet 10 (total – 21 points) till January 11, 2016

Exercise 10-1 (5 points)

For the following distribution functions check whether they are infinitely divisible.
1. (1 point) Gaussian distribution.
2. (1 point) Poisson distribution.
3. (1 point) Gamma distribution.
4. (1 point) Uniform distribution.
5. (1 point) Beta distribution.

Exercise 10-2 (3 points)

Find parameters (a, b, ν) in the Lévy-Khintchin representation of a characteristic function for
1. (1 point) Gaussian distribution.
2. (1 point) Poisson distribution.
3. (1 point) Gamma distribution.

Exercise 10-3 (3 points)

Let $X_1, \ldots, X_n$ be a sequence of i.i.d. infinitely divisible random variables. Show that the random variable $Y = \sum_{k=1}^{n} a_i X_i$ is infinitely divisible as well for arbitrary $a_1, \ldots, a_n \in \mathbb{R}$ and calculate its Lévy characteristic.

Exercise 10-4 (3 points)

Let the non-negative integer-valued r.v. ξ be infinitely divisible. Express this property of ξ in terms of its probability generating function $g(s) = \mathbf{E}s^{\xi}, |s| \leq 1$.

Exercise 10-5 (3 points)

Show that the distribution of a bounded r.v. Z is infinitely divisible if and only if Z is constant.

Exercise 10-6 (4 points)

Show that the function $\varphi : \mathbb{R} \to \mathbb{C}$ given by

$$\varphi(z) = \exp(\psi(z)), \qquad \psi(z) = 2 \sum_{k=-\infty}^{\infty} (\cos(2^k z) - 1)$$

is the characteristic function of an infinitely divisible random variable.

Hint: Use the Lévy-Khintchine representation with Lévy measure $\nu(\pm 2^k) = 2^{-k}, k \in \mathbb{Z}$.