Stochastics II
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Exercise sheet 2 (total – 18 points) till November 02, 2016

Exercise 2-1 (4 points)

Let $\xi_k, k \in \mathbb{N}$ be independent random variables defined on some probability space $(\Omega, \mathcal{F}, \mathbf{P})$ and having geometric distribution with parameter $p \in (0,1) : \mathbf{P}[\xi_i = k] = p(1-p)^{k-1}, k \in \mathbb{N}, i \in \mathbb{N}$.

Let $S_n = \xi_1 + \ldots + \xi_n$ and $X_i = \begin{cases} 1, & \text{if there is } n \in \mathbb{N} \text{ such that } S_n = i, \\ 0, & \text{otherwise.} \end{cases}$

Let also $Y_k, k \in \mathbb{N}$ be independent Bernoulli random variables with $\mathbf{P}[Y_i = 1] = p, \mathbf{P}[Y_i = 0] = 1 - p$ defined on some other probability space $(\Omega^0, \mathcal{F}^0, \mathbf{P}^0)$. Show that the stochastic processes $\{X_i, i \in \mathbb{N}\}$ and $\{Y_i, i \in \mathbb{N}\}$ have the same finite-dimensional distributions.

Exercise 2-2 (6 points)

Suppose that a random process $\{X(t), t \in [0,1]\}$ has continuous trajectories. Prove that the following sets are measurable.

1. $\{\omega \in \Omega | \min_{t\in[0,1]} X(t,\omega) < 1\}$,
2. $\{\omega \in \Omega | \exists t \in [0,1) : X(t,\omega) = 1\}$,
3. $\{\omega \in \Omega | X(t,\omega), t \in [0,1]$ is non-decreasing$\}$.

Exercise 2-3 (2 points)

Prove that stochastic process $\{X(t), t \in \mathbb{R}_+\}$ is measurable assuming its trajectories are: (a) right continuous; (b) left continuous.

Exercise 2-4 (3 points)

Suppose that all random variables $X(t)$ of a stochastic process $\{X(t), t \in \mathbb{R}_+\}$ are independent and uniformly distributed on $[0,1]$. Prove that the process is not continuous in probability.

Exercise 2-5 (3 points)

Prove that Poisson process is stochastically continuous although it does not posses any a.s. continuous modification.