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Exercise sheet 4 (total – 18 points) till November 16, 2016

Exercise 4-1 (4 points)

Let $\{X(t) = f(t-\tau), t \in \mathbb{R}_+\}$ where τ is a $Exp(\lambda)$-distributed random variable and $f(x) = (1-|x|)\mathbb{I}\{|x| \leq 1\}, x \in \mathbb{R}$. Is the process X mean square differentiable? If so, find the derivative X', defined in L_2 sense, and $\mathbf{E}(X'(1))^2$.

Exercise 4-2 (3 points)

Prove that if $\{X(t), t \in \mathbb{R}_+\}$ is a mean square continuous process then the process $Y(t) = \int_0^t X(s)ds$ is mean square differentiable and $Y'(t) = X(t), t \in \mathbb{R}_+$

Exercise 4-3 (2 points)

Prove that the Wiener process has a continuous modification.

Exercise 4-4 (5 points)

Prove that the Wiener process is not stochastically differentiable.

Exercise 4-5 (4 points)

Prove the following statements.
1. The Wiener process possesses independent increments.
2. The Poisson process with intensity λ has independent increments.

Hint: Find the characteristic function of increments and verify the formula (1.7.1) from the lecture notes.